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Railway Traffic Rescheduling with Minimization of Passengers' Discomfort

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ABSTRACT

Optimization models for railway traffic rescheduling in the last decade tend to develop along two main streams. One the one hand, train scheduling models strives to incorporate any relevant detail of the railway infrastructure having an impact on the feasibility and quality of the solutions from the viewpoint of operations managers. On the other hand, delay management models focus on the impact of rescheduling decisions on the quality of service perceived by the passengers. Models in the first stream are mainly microscopic, while models in the second stream are mainly macroscopic. This paper aims at merging these two streams of research by developing microscopic passenger-centric models, solution algorithms and lower bounds. Fast iterative algorithms are proposed, based on a decomposition of the problem and on the exact resolution of the subproblems. A new lower bound is proposed, consisting of the resolution of a set of min-cost flow problems with activation constraints. Computational experiments, based on a real-world Dutch railway network, show that good quality solutions and lower bounds can be found within a limited computation time.

Keywords: Train Scheduling, Delay Management, Passenger Routing, Mixed-Linear Integer Programming, Min-Cost Flow.

1. Introduction

In the last years, many railway companies, at least in Europe, are experiencing increasing difficulties to face the ever increasing transport demand while ensuring good quality of service (QoS) to the passengers, also due to the limited space and funds to build new infrastructure in bottleneck areas. These facts stimulated the interest of practitioners and theoreticians for new effective approaches to railway traffic rescheduling aiming at the reduction of delays of trains and passengers. As a matter of fact, the literature on this subject experienced a significant growth in the last years, see, e.g., Caprara et al. (2006), Cacchiani et al. (2014) and Binder et al. (2014). The existing models can be classified according to different criteria. Microscopic and macroscopic representations differ for the level of detail of the railway infrastructure, the first case aiming at including in the model all the relevant details of the railway infrastructure having an impact on railway traffic rescheduling and the second limiting the description at higher level. Operations-centric models focus on the minimization of railway operations objectives, such as train delays, whereas passenger-centric models focus on the maximization of the QoS perceived by the passengers.

The recent trend in the railway traffic rescheduling literature is to incorporate an increasing level of detail and realism in the models while keeping the computation time of solution algorithms compatible with real-time applications, or by taking into account the impact of rescheduling decisions on the QoS perceived by the passengers. However, most papers focus on microscopic operations-centric models or macroscopic passenger-centric models. This paper aims at merging these two lines of research by developing microscopic passenger-centric models. Following the most common terms adopted in the literature, in this paper we refer to microscopic operations-centric models as *train scheduling* (TS) models, to macroscopic passenger-centric models as *delay management* (DM) models and to microscopic passenger-centric models as *microscopic delay management* (MDM) models.

With respect to the existing literature, we move a step forward towards the explicitly consideration of passenger flows within a new microscopic optimization model for railway traffic rescheduling. We also propose methods for the fast computation of upper and lower bounds. Empirical evaluation shows that good quality solutions can be found within limited computation time for a real-world railway network. The algorithms for the computation of the upper bounds are iterative methods based on the decomposition of the MDM model and on the solution of the decomposed models. We show that good quality lower bounds can be quickly computed by solving a set of suitable min-cost flow problems with activation constraints.

The paper is organized as follows. Section 2 reviews the relevant literature on railway traffic rescheduling, focusing on TS, DM and the few approaches facing the combined MDM. Section 3 introduces the relevant definitions and notations, later used in Section 4 to describe the MDM model. Section 5 presents the considered lower bounds, while Section 6 illustrates the algorithms to compute upper bounds. Section 7 shows the behaviour of the algorithms through a numerical example. Section 8 reports on the computational experiments on a rail Dutch network, which demonstrate the potential of the proposed approaches. Some conclusions and directions for future research are reported in Section 9.

2. Literature review

This section discusses the literature on railway traffic rescheduling with specific consideration to the TS models, the DM models and the few existing approaches for MDM. The first stream of models focuses on the operations, and follows the typical performance indicators of railway traffic controllers. The second stream focuses on passenger-centric approaches with a simplified view of the railway infrastructure. In the third stream we review some recent approaches aiming to combine TS and DM models.

2.1. Train scheduling

The complexity of the TS problem stems from the limited overtaking capacity of railway lines and from the constraints of the safety system, caused by the signal status and speed restrictions. The competition of trains for the available capacity can be modelled only when a sufficient level of detail is considered. One of the most effective approaches to tackle such complexity is based on the blocking time theory (see, e.g., Hansen and Pachl 2014) and on the alternative graph formulation of the blocking job shop scheduling model with no-swap constraints (Mascis and Pacciarelli 2002). This problem has been shown to be NP-hard even in the feasibility case (Mascis and Pacciarelli 2002). Advanced scheduling approaches based on the alternative graph model are able to quickly solve real-life instances in which train arrival times, orders and routes, are considered variable (see e.g. D'Ariano et al. 2007, D'Ariano et al. 2008, Mannino and Mascis 2009, Corman et al. 2010). Remarkable improvements with respect to the current practice and/or to the basic dispatching rules adopted in most practical applications are reported in these papers. Other approaches to TS based on Mixed Integer Linear Programs (MILPs) are reported, e.g., in Törnquist Krasemann (2011), Lamorgese and Mannino (2014), Meng and Zhou (2014), Pellegrini et al. (2014). The goal of all these approaches is to find a *train schedule*, i.e., a departure/passing time for each train et each relevant point of the railway network, compatible with the real time position of each train and such that a suitable function of the train delays is minimized. Such goal can be achieved by all approaches in practical size networks and within a computation time compatible with real-time operations.

One weakness of all these models is the limited view of passenger needs and expectations, which are taken into account only indirectly, e.g., by penalizing train delays. Among the works trying to enlarge the scope of these approaches, Corman et al. (2011) propose an iterated lexicographic optimization of train delays, given a division of trains into classes. The delay of each class is minimized provided that the delay of higher priority classes does not increase. This approach might be applied by defining priority classes according to the estimated importance of particular trains for the overall passenger QoS. Espinosa-Aranda and Garcia-Rodenas (2013) extend the alternative graph model to take into consideration passenger flows at some extent. They minimize the weighted train delay by an heuristic algorithm which generalizes the AMCC rule of D'Ariano et al. (2007). By weighting each train with an off-line estimation of the number of passengers onboard the train one can take into account the passenger QoS. However, other sources of passenger discomfort such as broken transfer connections are not taken into account. A bi-objective optimization approach, trying to directly take into account the effect of broken transfer connections within a TS model, is proposed by Corman et al. (2012). A weight is associated to each passenger connections, depending on its importance for the passenger QoS, and then the Pareto frontier is computed where the two objective functions are the train delays and the total weight of broken connections.

2.2. Delay Management

A different stream of research directly faces the optimization of the QoS perceived by the passengers, and is based on the concept of customer-oriented dispatching introduced in Suhl et al. (2001) and Schöbel (2001). Among this stream of research, Schöbel (2007) defines the delay management (DM) problem as the problem of deciding whether to keep or not transfer connections during operations, a decision that directly affect passenger QoS. Dollevoet et al. (2012) enlarge the scope of DM to include the possibility of rerouting passengers in order to reduce their travel time. The approaches in this stream of research are currently based on macroscopic models, i.e. they consider only arrival and departure at stations, neglecting the actual capacity of platforms, interlocking areas and block sections. As a result, the promised passenger delay is only a lower bound of the one achievable in practice, and even the proposed solution might turn to be infeasible when implemented in practice. To overcome this drawback, more recent works have tried to include some capacity constraints in the models, though in approximated form along lines (Schachtebeck and Schöbel 2010) and at stations (Dollevoet et al. 2014A). Though the trend in the literature on DM is for the inclusion of an increasing level of detail in the models, a MDM model with passenger routing is still missing.

2.3. Microscopic delay management

Very recently, research has started addressing the interaction of passenger flows with train scheduling. Tomii et al. (2005) first introduce the minimization of passenger dissatisfaction as an objective of microscopic railway traffic rescheduling. Takeuchi et al. (2006) propose robustness indices to classify dispatching actions based on the impact they have on passengers. Sato et al. (2013) address the problem of minimizing passenger inconvenience on simple railway lines, taking into account disruptions that might require adjustment of vehicle schedules. The passenger inconvenience is divided into three components, i.e., the time spent onboard the trains, the waiting time at platforms and the number of transfers. The approach results in a MIP problem that is solved by commercial software. They report in general that trains can decrease passengers inconvenience by increasing delays (i.e. for catching more passengers at a busy station). Control actions for railway lines without rerouting of passengers are studied; optimization variables are the orders of departure of trains along a line, and platforms assignment at stations. Passenger flows are routed along path of minimum inconvenience, and then considered fixed while the timetable is adjusted to a minor extent. Capacity at stations is approximated by means of headway times along the line.

A combination of the delay management approach with the microscopic models based on the alternative graph concept is proposed by Dollevoet et al. (2014B), in which passengers delays are optimized by iteratively solving a microscopic train scheduling problem (without knowledge of passenger flows) and a macroscopic delay management problem (without explicit modelling of limited infrastructure capacity). Passengers are always assigned to a shortest path between their origin and destination. The procedure delivers in few iterations good feasible solutions.

Binder et al. (2014) propose the time spent by the passengers in the system as the most important indicator for passenger discomfort and point out the need for further research to evaluate how close a heuristic solution is to an optimal solution in terms of passenger satisfaction. To this aim, the introduction of an overall optimization framework is needed.

2.4. Paper contribution

This paper addresses several issues that have not been fully addressed by previous research. We integrate microscopic railway traffic rescheduling and passenger point of view into a single MILP. We consider the minimization of the time spent by the passengers in the system as objective function. Passenger flows are modelled based on Origin-Destination description, with possibility of transfer connections and rerouting. A new good-quality lower bound is proposed which can be quickly computed by solving a set of suitable min-cost flow problems. Some fast heuristic algorithms are proposed, based on the iterated resolution of TS and DM models. Computational experiments on a real-world Dutch network allow us to evaluate the quality of lower and upper bounds. Comparison with the optima, when available, or with lower bounds on the optima demonstrates the potential of the proposed approaches.

Differently from Dollevoet et al. (2014B) and Sato et al. (2013), scheduling and traffic control decisions are taken by a single model, so that we can evaluate the optimality gap of heuristic solutions. Moreover, we focus on fast heuristics, able to provide microscopically feasible solutions within a short computation time.

3. MDM problem definition

We address the problem of computing in real time a schedule for all trains and a routing for each passenger so that passengers have the least possible discomfort. In general, discomfort can be related to the time spent in various conditions (Wardman 2004). However, according to Binder et al. (2014), in this paper we adopt the passenger travel time as a surrogate for the passenger discomfort.

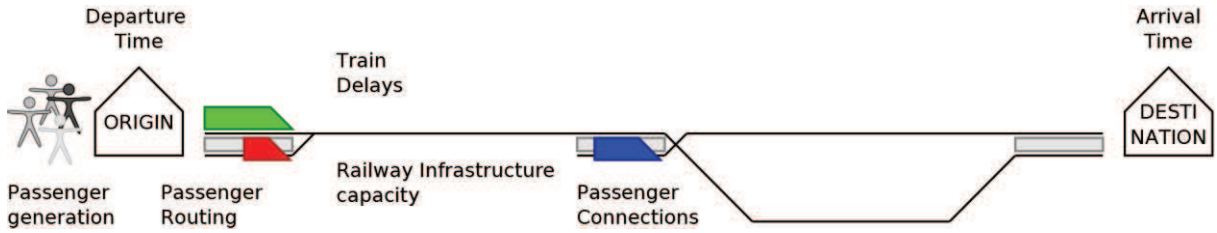

Figure 1. Pictorial representation of the elements of the MDM problem

We refer to Figure 1 for a graphical summary aiming at considering all aspects of the model. Passengers start from an origin station at given arrival times and want to reach a destination station as soon as possible. We discretize the arrival time of passengers, and refer to a group of passengers arriving at the same origin o at the same time w and having the same destination d as a *demand odw*. We call *passenger generation* the definition of all demands. Moreover, we assume the trains having sufficient capacity to accommodate all passengers in each group, so that all the passengers in each demand share the same route from origin to destination. Note that this assumption is not particularly restrictive, since exceedingly large groups of passengers can be split into multiple demands.

We call *passenger routing* the definition of a route for each demand from the associated origin to the associated destination stations, possibly including transfers between pairs of connected trains at intermediate stations. In our model we assume that passengers are rational and informed, and that each demand aims at reaching the destination within the shortest time from the arrival at origin station. To this aim, transfers between trains are used by passengers whenever convenient, i.e., each demand follows the shortest route to destination given the train schedule. We call a *passenger connection* the transfer of a demand from a feeder train to a connected train at an intermediate station along the demand route. The possibility of a passenger connection depends on train scheduling decisions, since there must be sufficient time between the arrival time of the feeder train and the departure time of the connected train to allow passenger transfer.

Train scheduling decisions are necessary whenever *train delays* occur that make infeasible the plan of arrivals/departures described by the timetable. Such decisions must take into account the limited *capacity* of the *railway infrastructure*, which limits the possibility of rescheduling train movements. This capacity depends on railway safety regulations, which make use of the signalling system to limit the speed of each train depending on the position of other trains in the network. More formally, with the so-called fixed-block railway regulation the railway network is partitioned in track segments, called block sections, separated by signals. When a block section is occupied by a train, the signal aspect at its entrance forces other incoming trains to stop before the signal, while the signal aspect at the entrance of the previous block section forces incoming train to reduce their speed. Therefore, a train can run at high speed only if the two block sections ahead are empty while it can run at reduced speed if only the block section ahead is empty. Moreover, each block section can host at most one train at a time. By viewing the block sections as machines of single capacity and the trains as jobs, the combinatorial structure of the TS problem is similar to the blocking job shop scheduling problem with no-swap constraints described by Mascis and Pacciarelli (2002), where the processing time of an operation correspond to the *running time* necessary to traverse a block section or to the *dwell time* at the platform of a scheduled stop. However, in the TS problem there are additional constraints. For example, a train cannot depart from a station before its scheduled departure time and its departure from a station can be constrained to be sufficiently larger than the arrival of another (feeder) train so that the passengers can move from the latter to the former. Moreover, a train is late when its arrival time at a station is larger than the scheduled arrival time.

Delays can be caused either by external disturbances or by the propagation of delays from one train to the other due to the many constraints of the TS problem. Specifically, primary delays are due to external disturbances that can be recovered only to a certain extent by exploiting running time supplements of the timetable. Secondary delays are determined by rescheduling decisions in response to primary delays, and are the result of a delay propagation due to some conflicts. A *conflict* occurs whenever two trains require the same block section at the

same time. The conflict is typically solved by specifying a passing order for the trains at the block section. Rescheduling decisions typically result in extra time required by trains to run between two stations, or to dwell at stations. Determining the train order and the entrance time of each train in each block section is the main subject of the TS problem. More detailed descriptions of the TS definition and formulation can be found, e.g., in D'Ariano et al. (2007, 2008) and Corman et al. (2009, 2014B).

In this paper we deal with *timetable perturbations*, i.e., with primary delays that can be managed without the need for train cancellations or rerouting. Microscopic train scheduling models for traffic disruptions, arising in case of huge train delays and/or network failures, are studied for instance by Almodovar and Garcia Rodenas (2013) and Corman et al. (2014A).

4. Mathematical model

In this section a mathematical program for the MDM problem is introduced, which incorporates passenger flows in a microscopic train rescheduling model. Most optimization models for train rescheduling are based on job shop scheduling models, and associate trains to jobs and block sections to machines. Those models exploit a time-continuous formulation for train movements, and binary decision variables for the decision of orders of trains at shared block section. In the recent years, the alternative graph model (Mascis and Pacciarelli, 2002) has become very popular to represent railway operations in a job shop scheduling formulation, starting from the work of D'Ariano et al. (2007). Based on this stream of research, in this paper we start from the MILP version of the train rescheduling model of D'Ariano et al. (2007) and add passenger flows to obtain a microscopic delay management model.

4.1. Job shop scheduling models for microscopic train rescheduling

This section recalls the train rescheduling model of D'Ariano et al. (2007). The problem of controlling railway traffic with microscopic detail corresponds to a job shop scheduling problem with blocking no-swap constraints (Mascis and Pacciarelli 2002). Blocking no-swap constraints model the so-called fixed-block railway regulation that a train on a given block section cannot move forward if the block section ahead is not available or if it is occupied by another train. We next recall very briefly the alternative graph model (Mascis and Pacciarelli, 2002), which is the basis of the train rescheduling model of D'Ariano et al. (2007).

An alternative graph G is a triple (N, F, A). Nodes in N correspond to operations, each associated to the occupation of a block section by a train, representing either the traversing of a block section or the dwell in a station where a train has a planned stop. For each operation $i \in N$, a continuous variable h_i is associated to its starting time. A dummy operation 0, with $h_0 = 0$, is used to represent the starting of the scheduling horizon.

Arcs in the set F model time relations between the starting times of some pairs of operations. For example, if i and j are associated to the traversing of two consecutive block sections by the same train then the directed arc (i,j) models the fact that h_j must be greater or equal to h_i plus the minimum running time of the train on the block section associated to i. In this case the minimum running time p_{ij} is a weight on the arc (i,j). Fixed arcs can also model a minimum departure time of a train from a block section, which in this case is modelled with an arc $(0,j)$ having weight p_{0j} equal to the minimum departure time. Other examples will be discussed later in this section. In all cases, however, an arc $(i, j) \in F$ corresponds to the constraint $h_j \geq h_i + p_{ij}$.

Arcs in the set A model potential conflicts between trains on shared resources. Whenever two trains claim the same infrastructure element (block section, platform, etc.) at the same time, a conflict arises and a decision on the order of the two trains on the infrastructure element must be then taken to resolve the conflict. In order to take into account signal status, a minimum time separation between the starting times of the conflicting operations is needed to ensure that the second train enters the infrastructure element after the first train has left it (i.e., the first train is occupying the next infrastructure element). Formally, let i and j be consecutive operations

of a train, k and l be consecutive operations of the other train, and let i and k be the two conflicting operations associated to the entrance of the two trains in the same block section. Then, the ordering decision is modeled with a pair of alternative arcs $\big((j,k),(l,i)\big) \in A$, representing the alternative constraints $\big(h_k \geq h_j + s_{jk}\big)$ OR $(h_i \geq h_l + s_{li})$, where s_{jk} is the minimum time separation when let i precedes k and s_{li} is the minimum time separation when let k precedes i. The set A thus contains a pair of alternative arcs for each pair of potentially conflicting operations. Finding a feasible solution for a given alternative graph G consists of selecting one arc for each pair in A (which corresponds to defining an order to each pair of conflicting operations), thus obtaining a set S of selected arcs from A. The selection S is feasible if and only if the graph $(N, F \cup S)$ contains no positive length cycles. More details on job shop scheduling based models for railway traffic can be found in D'Ariano et al. (2007) or Corman et al. (2011).

4.2. Passenger routing model

The passenger routing problem (or passenger assignment problem) studies the distribution of passengers onto the railway network. We consider a time discrete model for passenger arrivals at each station. Hence, we assume to know the number of passengers willing to reach the same destination $d \in D$ from the same origin $o \in O$, starting their journey at the same time w, for a discrete set of arrival times W. The discrete model is justified by the observation that all the passengers with the same destination arriving at a station between two consecutive train departures will move together in the network as a group, due to the assumptions that each train has infinite capacity and each passenger aims at reaching his/her destination in the minimum time. In this paper, we refer to a group of passengers going from o to d and arriving in o at time w as a triple odw, hereinafter denoted as *demand*, and let ODW be the set of all demands odw. Note that, once the train schedule is fixed, each demand odw moves in the network independently from the other triples, i.e., the choice of a particular routing for a given odw does not influence the routing of any other odw. Moreover, we assume that all passengers in a demand odw will follow the same OD path (we assume this path as unique, possibly breaking ties arbitrarily). Hence, the passenger routing aspect of the MDM problem is similar to a multicommodity flow problem on graph $(N, F \cup S)$, in which a commodity is associated to each odw triple. The only difference is that passengers may change train only at scheduled stops if a connection exists, i.e., only if the connected train departs from the station sufficiently later that the arrival of the passengers. To take into account this difference, we introduce a set of *connection* arcs C, each associated to a pair (i,j) of operations, where i is the operation associated to the arrival of the feeder train at the station and j is the operation associated to the departure of the connected train from the station. Each arc has a weight c_{ij} equal to the minimum time for transferring passengers from the feeder train to the second one. Each arc in $(i,j) \in C$ is active only if $h_j \geq h_i + c_{ij}$. Therefore, the passenger routing aspect of the MDM problem can be modelled as a multicommodity flow problem where passengers may flow only through the arcs of F and through the active arcs of C.

4.3. Microscopic delay management model

We next introduce a MILP model for the MDM problem combining train rescheduling and passenger routing decisions. To this aim we a introduce a new disjunctive graph $G^P = (N \cup N^{ODW}, F \cup F^{ODW} \cup C, A)$ where N, F and A are the sets defined in Section 4.1, while N^{ODW}, F^{ODW} and C are sets of nodes, fixed arcs and connection arcs that are necessary to take into account passenger routing. The set N^{ODW} contains two nodes for each demand $odw \in ODW$, namely a source node $start^{odw}$ with supply equal to 1 and a sink node end^{odw} with demand equal to 1. These nodes take into account the origin and destination of the flow associated to odw. Fixed arcs in F^{ODW} link arriving/departing passengers of odw to the first/last train they may take on their journey. By letting δ_{start}^{odw} be the set of nodes associated to train departures from the origin station of odw ad by letting δ_{end}^{odw} be the set of arrivals at the destination station of odw, F^{ODW} is the set of arcs $(start^{odw}, j)$ with $j \in \delta_{start}^{odw}$ plus the arcs (i, end^{odw}) with $i \in \delta_{end}^{odw}$. C is the set of connection arcs defined in Section 4.2. There is an arc $(i,j) \in C$ for each pair of nodes i, associated to a train arrival, and j, associated to a train departure, at/from the same station, for all stations. The connection is active if $h_j \geq h_i + c_{ij}$, i.e., if a passenger can transfer from the feeder to the connected train.

In order to translate G^P into a MILP model, we can simply formulate each disjunction as a *big-M* linear constraint using binary variables. We need two kinds of binary variables: x_{ijkl} associated to the choice of an alternative arc in the pair $\big((j,k),(l,i)\big) \in A$ and q_{ij}^{odw} associated to the use of connection $(i,j) \in C$ by the passengers on odw and to the assignment of passengers to arcs in $F \cup F^{odw}$. Besides binary variables the model makes use of real variables h_i, equal to the starting time of each operation in $N\backslash\{0\}$, and T_{odw}, equal to the arrival time at destination of the passengers in odw.

In order to describe the MDM formulation, we next summarize all the notation used.

Parameters

- $F = \{(i,j), \dots\}$ is the set of fixed directed arcs representing running, departure, arrival, dwell, origin, destination of each train
- $C = \{(i,j), \dots\}$ is the set of connection directed arcs between train arrivals and departures at a station
- $A = \{((j,k),(l,i)), \dots\}$ is the set of pairs of alternative directed arcs representing train ordering decisions
- p_{ij} is the weight on fixed arcs
- s_{ki} is the weight on alternative arcs
- c_{ij} is the weight on connection arcs
- ODW is the set of demands, i.e., groups of passengers willing to move from *o* to *d* starting at time *w*
- $start^{odw}$ is the origin node for passengers in $odw \in ODW$
- end^{odw} is the destination node for passengers in $odw \in ODW$
- δ_{start}^{odw} is a set of nodes, each associated to the departure of a train from the origin station of *odw*
- δ_{end}^{odw} is a set of nodes, each associated to the arrival of a train at the destination station of *odw*
- δ_i^{out} is the set of arcs in $F \cup F^{ODW} \cup C$ outgoing node *i* that can be used by passengers
- δ_i^{in} is the set of arcs in $F \cup F^{ODW} \cup C$ ingoing node *i* that can be used by passengers
- F^{ODW} is the set of arcs $(start^{odw}, j)$ with $j \in \delta_{start}^{odw}$ plus the arcs (i, end^{odw}) with $i \in \delta_{end}^{odw}$
- π_{odw} is the arrival time at origin station $o \in O$ of demand odw, i.e., the starting time of $start^{odw}$
- n_{odw} is the number of passengers for demand odw

Variables

- h_i is the starting time of operation $i \in N\backslash\{0\}$
- x_{ijkl} equal to 1 if arc (l,i) is selected from pair $((j,k),(l,i)) \in A$, i.e., if $h_i \geq h_l$ (and thus $h_i \geq h_k$ since $(k,l) \in F$), and 0 otherwise
- q_{ij}^{odw} equal to 1 if arc $(i,j) \in F \cup F^{ODW} \cup C$ belongs to the path of odw, and 0 otherwise
- T_{odw} arrival time at end^{odw} of passengers in odw

The MDM formulation is as follows.

$$\min \sum_{odw \in ODW} n_{odw}\,(T_{odw} - \pi_{odw})$$

$$h_j \geq h_i + p_{ij} \qquad \forall (i,j) \in F \qquad \text{(A1)}$$

$$h_i \geq h_l + s_{li} - M(1 - x_{ijkl}) \qquad \forall ((j,k),(l,i)) \in A \qquad \text{(A2)}$$

$$h_k \geq h_j + s_{jk} - M\, x_{ijkl} \qquad \forall ((j,k),(l,i) \in A \qquad \text{(A3)}$$

$$\textstyle\sum_{j \in \delta_{start}^{odw}} q_{start,j}^{odw} = 1 \qquad \forall\, odw \in ODW \qquad \text{(A4)}$$

$$\textstyle\sum_{i \in \delta_{end}^{odw}} q_{i,end}^{odw} = 1 \qquad \forall\, odw \in ODW \qquad \text{(A5)}$$

$$\sum_{(i,j)\in\delta_i^{out}} q_{ij}^{odw} = \sum_{(i,j)\in\delta_i^{in}} q_{ij}^{odw} \qquad \forall\, i \in N \setminus \{0\}, \forall\, odw \in ODW \qquad \text{(A6)}$$

$$h_j \geq \pi_{odw} + M\left(q_{start,j}^{odw} - 1\right) \qquad \forall\, j \in \delta_{start}^{odw}, \forall\, odw \in ODW \qquad \text{(A7)}$$

$$h_j \geq h_i + c_{ij} - M(1 - q_{ij}^{odw}) \qquad \forall (i,j) \in C, \forall\, odw \in ODW \qquad \text{(A8)}$$

$$T_{odw} \geq h_i - M\left(1 - q_{i,end}^{odw}\right) \qquad \forall\, i \in \delta_{end}^{odw},\ \forall\, odw \in ODW \qquad \text{(A9)}$$

$$h_i \geq 0 \qquad \text{(A10)}$$

$$x_{ijkl}, q_{ij}^{odw} \in \{0,1\} \qquad \text{(A11)}$$

We next briefly describe the model. The objective function is the minimization of the total time spent in the system by all passengers. Constraints (A1) relate to fixed arcs, e.g., to running, dwell, and departure operations. Constraints (A2)-(A3) model the choice of an arc from each pair of alternative arcs in set A, i.e., they relate to the choice of orders between trains over each infrastructure element with limited capacity. Constraints (A4)-(A6) define a min-cost flow problem to assign a path from o to d to the passengers in each triple odw. Specifically, (A4) and (A5) describe the departure of passengers from the origin station and the arrival at destination, respectively, while (A6) model the typical flow balance constraints at intermediate nodes.

Constraints (A7)-(A9) model the interaction between train and passenger arrival/departure times. Specifically, inequalities (A7) constrain a train carrying passengers in odw to depart from o after the arrival time of the passengers, (A8) take into account passengers transfer from a train to another and force the connected train to depart sufficiently later than the arrival of the feeder train, constraints in (A9) allow the computation of the passengers arrival time at destination. Finally, (A10)-(11) define type and bounds for the optimization variables used.

5. Lower Bounds

This section introduces lower bounds to the MDM problem which can be used to validate the quality of heuristic solutions.

5.1. Macroscopic delay management

The first lower bound is based on the relaxation of constraints (A2)-(A3) that represent train sequencing decisions on the microscopic infrastructure of the network. The relaxed problem is then equivalent to a macroscopic delay management problem with rerouting of passengers, over an infinite capacity infrastructure, as reported in Dollevoet et al. (2012).

5.2. Single OD lower bound

The second lower bound is a relaxation of the first one which neglects the interactions between different *OD* pairs. Each *OD* pair is evaluated independently from the others by exploiting all the routing alternatives available to reach a destination $d \in D$ from an origin $o \in O$ in the given time horizon. Therefore, in what follows, all the demands *odw* considered in the lower bound computation share the origin and destination and differ only for the arrival time *w*. We name **train combination** a sequence of trains and passenger transfers, that allows to reach the destination *d* starting from the origin *o*. Each combination *j* has a minimum departure time τ_j, equal to the earliest departure time of the train leaving *o*, and can be delayed by any amount of time if useful to reduce the travel time of the passengers travelling from *o* to *d*. For each combination, the running time r_j is also given, that we assume to be constant even if the train departure is delayed, since each train disregards the presence of other trains in the network. Note that, only potentially promising train combinations have to be considered in the model. In fact, for each train leaving *o*, only the combination reaching *d* with minimum travel time is relevant to compute a lower bound. Therefore, the number of combinations to be considered is equal to the number of trains departing from *o* in the given time horizon.

The lower bound is obtained by solving the problem of assigning all the demands odw associated to the same origin o and destination d to the available train combinations connecting o and d. In what follows, we order the demands associated to the pair od for increasing arrival time, and use the index i to denote the i-th demand associated to the pair od. Therefore, π_i and n_i denote the arrival time and the number of passengers of the i-th demand. Clearly, the departure time of the train combination j is the maximum between τ_j and the arrival time π_i of the latest demand assigned to j. The travel time of the i-th demand assigned to combination j is then equal to r_j plus the difference between the departure time of j and π_i. As a consequence of the above discussion, in the optimal solution of this relaxation it is sufficient to consider only a limited set of departure times for each combination, namely τ_j and all the values $\pi_i \geq \tau_j$. Based on this property we formulate the relaxed problem for a given pair od as a uncapacitated min-cost flow problem with additional constraints on the following digraph $(N_1 \cup N_2, A_1 \cup A_2)$.

Nodes in N_1 are associated to the demands i=1,…,W and nodes in N_2 are associated to the train combinations j=1,…,J. We assume the demands/combinations are numbered for increasing arrival/departure times, i.e., $i > h$ implies $\pi_i \geq \pi_h$ and $j > k$ implies $\tau_j \geq \tau_k$. There are two sets of direct arcs: *waiting* arcs in A_1 between pairs of consecutive nodes in N_1 and *travelling* arcs in A_2 between a node in N_1 and a node in N_2. More precisely, there is a waiting arc (i,i+1) between the i-th demand with the next one, for i=1,…,W-1, having weight $\gamma_{i,i+1} = \pi_{i+1} - \pi_i$. For each pair ($i$-th demand, combination j) such that $\pi_i \leq \tau_j < \pi_{i+1}$ there is an arc (i,j)$\in A_1$ having weight $\gamma_{ij} = \tau_j - \pi_i + r_j$. For each pair ($i$, j) such that $\pi_i > \tau_j$ there is an arc (i,j) $\in A_2$ having weight $\gamma_{ij} = r_j$. Each node $i \in N_1$ is a source node with supply n_i, while each node $j \in N_2$ is a sink node accepting any non-negative flow.

The formulation of the problem includes two sets of variables: the (real) flow variables $u_{ij} \geq 0$, (i,j)$\in A_1 \cup A_2$, and the (binary) activation variables $v_{ij} \in \{0,1\}$, (i,j)$\in A_2$, which are required to express the constraint that for each train combination exactly one departure time must be chosen, i.e., only one arc ingoing each $j \in N_2$ must be activated. The objective function is the minimization of the flow cost $\sum_{(i,j)\in A_1 \cup A_2} \gamma_{ij} u_{ij}$. The constraints are associated to:

- the flow conservation in each node $i \in N_1$, $u_{i,i+1} - u_{i-1,i} + \sum_{(i,j)\in A_2} u_{ij} = n_i$;
- the activation of each arc (i,j) $\in A_2$, $u_{ij} \leq M v_{ij}$, with $M = \sum_{i\in N_1} n_i$;
- the choice of one arc (i,j) $\in A_2$ingoing each node $j \in N_2$, $\sum_{i:(i,j)\in A_2} v_{ij} = 1$.

The formulation is then as follows.

Variables

u_{ij}: if the i-th demand is assigned to combination j, u_{ij} is the number of passengers travelling on this combination

$$v_{ij} = \begin{cases} 1 & \text{if the } i-\text{th demand is the latest demand assigned to combination } j \\ 0 & \text{otherwise} \end{cases}$$

Input data

W number of demands associated to OD

n_i number of passengers in the i-th demand, i=1,…,W

π_i arrival time of the i-th demand, $i \in W$

J number of train combinations associated to pair od

τ_j minimum departure time of train combination j=1,…,J

r_j running time of train combination j=1,…,J

γ_{ij} time spent by each passenger flowing on arc $(i,j) \in A_1 \cup A_2$

Formulation

$$\min \sum_{(i,j)\in A_1 \cup A_2} \gamma_{ij} u_{ij}$$

$$u_{i,i+1} - u_{i-1,i} + \sum_{(i,j)\in A_2} u_{ij} = n_i \qquad i \in N_1 \qquad \text{(B1)}$$

$$u_{ij} \le M v_{ij} \qquad (i,j) \in A_2 \qquad \text{(B2)}$$

$$\sum_{i:(i,j)\in A_2} v_{ij} = 1 \qquad j \in N_2 \qquad \text{(B3)}$$

$$u_{ij} \ge 0 \qquad (i,j) \in A_1 \cup A_2 \qquad \text{(B4)}$$

$$v_{ij} \in \{0,1\} \qquad (i,j) \in A_2 \qquad \text{(B5)}$$

Example

We next show the formulation (B1)-(B5) for a small numerical example with five demands and three combinations. Input data are shown in Table 1, the digraph is shown in Figure 2(a). Figure 2(b) shows the flows in an optimal solution. The three arcs in A_2 activated in the optimal solution are (1,a), (5,b) and (5,c), though no passenger takes combination c. Arcs in A_2 not depicted in figure are not activated.

demand	n_i	π_i
1	100	5
2	25	10
3	12	15
4	45	20
5	182	27

combination	τ_j	r_j
a	7	36
b	18	28
c	24	35

Table 1 Input data for an instance with 5 demands and 3 combinations

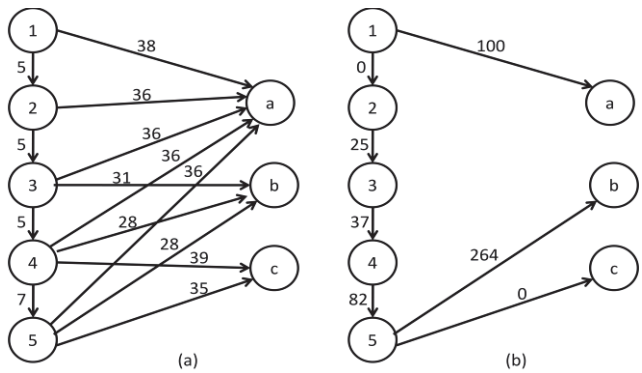

Figure 2: Formulation (a) and an optimal solution (b) for an example with five demands and three combinations

The MILP formulation for the example in Figure 2(a) is as follows.

$\min 5u_{12} + 5u_{23} + 5u_{34} + 7u_{45} + 38u_{1a} + 36u_{2a} + 36u_{3a} + 36u_{4a} + 36u_{5a} + 31u_{3b} + 28u_{4b} + 28u_{5b} + 39u_{4c} + 35u_{5c}$

s.t.

$u_{12} + u_{1a} = 100$

$u_{23} + u_{2a} - u_{12} = 25$

$u_{34} + u_{3a} + u_{3b} - u_{23} = 12$

$u_{45} + u_{4a} + u_{4b} + u_{4c} - u_{34} = 45$

$u_{5a} + u_{5b} + u_{5c} - u_{45} = 182$
$u_{1a} \leq 364 v_{1a}$
$u_{2a} \leq 364 v_{2a}$
$u_{3a} \leq 364 v_{3a}$
$u_{4a} \leq 364 v_{4a}$
$u_{5a} \leq 364 v_{5a}$
$u_{3b} \leq 364 v_{3b}$
$u_{4b} \leq 364 v_{4b}$
$u_{5b} \leq 364 v_{5b}$
$u_{4c} \leq 364 v_{4c}$
$u_{5c} \leq 364 v_{5c}$
$v_{1a} + v_{2a} + v_{3a} + v_{4a} + v_{5a} = 1$
$v_{3b} + v_{4b} + v_{5b} = 1$
$v_{4c} + v_{5c} = 1$
$u_{ij} \geq 0 \qquad \forall (i,j) \in A_1 \cup A_2$
$v_{ij} \in \{0,1\} \qquad \forall (i,j) \in A_2$

6. Upper bounds

In this section, three heuristics are described to compute a feasible solution to the MDM problem. All heuristics are based on the decomposition of the problem into two sub-problems that are solved separately, namely the train scheduling problem and the passenger routing problem.

6.1. Heuristic H1

The first heuristic H1 we consider is inspired by the last phase (reducing passenger inconvenience) reported in the work by Sato, Tamura, and Tomii (2013). The idea is to solve the train scheduling problem by simply fixing variables x according to the sequence prescribed by the timetable. In fact, the timetable prescribes a total order of the train departure times from each station (if necessary by breaking ties arbitrarily). Given this order we simply select x_{ijkl} equal to 1 whenever the departure time of the train associated to operations i and j precedes the departure of the train associated to operations k and l. This precedence holds for all the resources shared by the two trains until the first point where the two paths diverge, i.e., until an overtake can be performed. In other words, the timetable defines a set S of directed arcs such that $(l,i) \in S$ if and only if $((j,k),(l,i)) \in A$ and $x_{ijkl} = 1$. Then, the following simplified version of problem (A1)-(A11) is solved in which constraints (C2) replace constraints (A2)-(A3).

$$\min \sum_{odw \in ODW} n_{odw} \left(T_{odw} - \pi_{odw}\right)$$

$h_j \geq h_i + p_{ij}$	$\forall (i,j) \in F$	(C1)
$h_i \geq h_l + s_{li}$	$\forall (l,i) \in S$	(C2)
$\sum_{j \in \delta_{start}^{odw}} q_{start,j}^{odw} = 1$	$\forall\, odw \in ODW$	(C3)
$\sum_{i \in \delta_{end}^{odw}} q_{i,end}^{odw} = 1$	$\forall\, odw \in ODW$	(C4)
$\sum_{(i,j) \in \delta_i^{out}} q_{ij}^{odw} = \sum_{(i,j) \in \delta_i^{in}} q_{ij}^{odw}$	$\forall\, i \in N \setminus \{0\}, \forall\, odw \in ODW$	(C5)
$h_j \geq \pi_{odw} + M\left(q_{start,j}^{odw} - 1\right)$	$\forall\, j \in \delta_{start}^{odw}, \forall\, odw \in ODW$	(C6)
$h_j \geq h_i + c_{ij} - M(1 - q_{ij}^{odw})$	$\forall (i,j) \in C, \forall\, odw \in ODW$	(C7)
$T_{odw} \geq h_i - M\left(1 - q_{i,end}^{odw}\right)$	$\forall\, i \in \delta_{end}^{odw}, \forall\, odw \in ODW$	(C8)
$h_i \geq 0$		(C9)
$q_{ij}^{odw} \in \{0,1\}$		(C10)

The rationale for this heuristic is that the sequencing decisions x make the overall MDM problem very complex to solve, and it might be worthwhile to fix them offline. On the other hand, the departure times of the trains can

still be optimized to reduce the travel time of the passengers. One limit of H1 is that for large instances, model (C1)-(C10) is still difficult to solve, as it is shown in Section 8.

6.2. Heuristic H2

Heuristic H2 can be seen as a speed-up of H1. Starting from the sequence prescribed by the timetable, the train scheduling problem is first solved by fixing both variables x and h in the MDM problem (A1)-(A11). Note that, when a value $\bar{h}_i$ is given to each $i \in N$, the routing options for the passengers are strongly reduced. In fact, the connections that can be used are limited to those $(i,j) \in C$ such that $\bar{h}_j \geq \bar{h}_i + c_{ij}$, that we call *active* connections. We let C_{active} be the set of active connections. Also the set of trains eligible for the passengers in *odw* is restricted to those trains departing from *o* at time larger or equal to π_{odw}. We call $\beta_{start}^{odw} \subseteq \delta_{start}^{odw}$ the set of nodes associated to the departure of a train from the origin station of *odw* after π_{odw}, i.e., $j \in \beta_{start}^{odw}$ if and only if $(start^{odw}, j) \in F^{ODW}$. The passenger routing problem is then simplified as follows.

Parameters

- $\bar{h}_i$ is the starting time of operation $i \in N$ given by the solution to the train scheduling problem
- C_{active} is the set of active connection arcs, i.e., such that $(i,j) \in C$ and $\bar{h}_j \geq \bar{h}_i + c_{ij}$
- F_{active}^{odw} is the set of active arcs in F^{ODW}, i.e., such that $(start^{odw}, j) \in F^{ODW}$ and $\bar{h}_j \geq \pi_{odw}$ or $(i, end^{odw}) \in F^{ODW}$
- $\beta_{start}^{odw} \subseteq \delta_{start}^{odw}$ is the set of nodes associated to the departure of a train from the origin station of *odw* after π_{odw}, i.e., $j \in \beta_{start}^{odw}$ if and only if $(start^{odw}, j) \in F^{ODW}$ and $\bar{h}_j \geq \pi_{odw}$
- δ_{end}^{odw} is the set of nodes associated to the arrival of a train at the destination station of *odw*
- $\beta_i^{out} \subseteq \delta_i^{out}$ is the set of arcs in $F \cup F_{active}^{odw} \cup C_{active}$ outgoing node *i* that can be used by passengers
- $\beta_i^{in} \subseteq \delta_i^{in}$ is the set of arcs in $F \cup F_{active}^{odw} \cup C_{active}$ ingoing node *i* that can be used by passengers

$$\min \sum_{odw \in ODW} n_{odw} \left(T_{odw} - \pi_{odw}\right)$$

$\sum_{j \in \beta_{start}^{odw}} q_{start,j}^{odw} = 1$	$\forall\, odw \in ODW$	(D1)
$\sum_{i \in \delta_{end}^{odw}} q_{i,end}^{odw} = 1$	$\forall\, odw \in ODW$	(D2)
$\sum_{(i,j) \in \beta_i^{out}} q_{ij}^{odw} = \sum_{(i,j) \in \beta_i^{in}} q_{ij}^{odw}$	$\forall\, i \in N \setminus \{0\}, \forall\, odw \in ODW$	(D3)
$T_{odw} \geq \bar{h}_i - M\left(1 - q_{i,end}^{odw}\right)$	$\forall\, i \in \delta_{end}^{odw},\ \forall\, odw \in ODW$	(D4)
$q_{ij}^{odw} \in \{0,1\}$	$\forall\, i, j, odw$	(D5)

In this model, constraints (D1)-(D3) correspond to the routing options left from (A4)-(A6) once the variables h are fixed. Constraints (D4) correspond to (A9). The advantage of H2 is the strongly reduced computation time with respect to H1, since train departure times are simply computed by shifting all traffic ahead to account for the train delays. The disadvantage is that train departure times are fixed without taking passengers into account. Therefore, passengers have less routing options, and this fact may deteriorate the performance of H2.

6.3. Heuristic H3

Heuristic H3 is an iterative heuristic which aims at improving the performance of H2. It solves alternatively a train scheduling problem and a passenger routing problem until no improvement is possible. Starting from the solution of H2, a new instance of the train scheduling problem is solved by taking into account the number of passengers assigned to each train, in order to reduce passenger travel time. In order to introduce the problem, let us define a set $E \subseteq N$ of operations associated to the train arrivals at some scheduled stops, and let a_e be the scheduled arrival time of $e \in E$. Let also f_e be the number of passengers disembarking the train according to the

passenger routing solution, either because they have reached their final destination or because of a transfer. With this notation, the quantity $z_e = max\{0, h_e - a_e\}$ is the delay of the associated train at the associated station that is experienced by the f_e passengers disembarking the train. The new instance of the train scheduling problem aims at rescheduling trains for the purpose of reducing this delay for all passengers, i.e., the objective function is the minimization of $\sum_{e \in E} f_e\, z_e$. The train scheduling problem (E) is as follows.

$$\min \sum_{e \in E} f_e\, z_e$$

$h_j \geq h_i + p_{ij}$	$\forall (i,j) \in F$	(E1)
$h_i \geq h_l + s_{li} - M(1 - x_{ijkl})$	$\forall ((j,k),(l,i)) \in A$	(E2)
$h_k \geq h_j + s_{jk} - M\, x_{ijkl}$	$\forall ((j,k),(l,i)) \in A$	(E3)
$z_e \geq h_e - a_e$	$\forall e \in E$	(E4)
$z_e \geq 0$	$\forall e \in E$	(E5)
$h_i \geq 0$	$\forall i \in N \backslash \{0\}$	(E6)
$x_{ijkl} \in \{0,1\}$	$\forall ((j,k),(l,i)) \in A$	(E7)

In the model, constraints (E1)-(E3) are equivalent to (A1)-(A3) while (E4)-(E5) allow the computation of train delays *z*.

Figure 3 summarizes the behaviour of H3. Given a train scheduling solution, the DM problem (D1)-(D5) is solved to compute a passenger routing, and the two problems (E1)-(E7) and (D1)-(D5) are iteratively solved until one of the two problems finds the same solution obtained in the previous iteration.

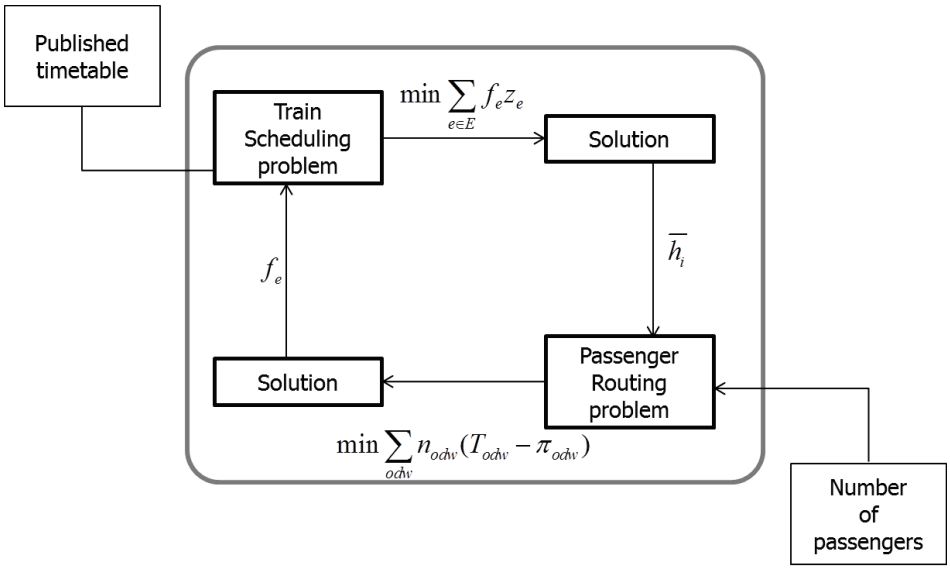

Figure 3: Flowchart of H3

7. Illustrative example

We next illustrate the application of the three heuristics on the small network shown in Figure 4. We consider 2 OD pairs, respectively A-D and B-D. For the OD pair A-D there are two demands in ODW, i.e., two groups of passengers (i.e., *w*=1,2), the first departing at time $\pi_{AD1} = 11$, with $n_{AD1} = 7$ passengers, and the second departing at time $\pi_{AD2} = 33$, with one passenger $n_{AD2} = 1$. The other OD pair has only one demand in ODW, departing at $\pi_{BD1} = 10$, with $n_{BD1} = 2$ passengers. A total of 10 passengers are thus in the network. Note that the signals are depicted in Figure 4 for the sake of illustration only. The precise position of signals does not affect the validity of the example, as far as a microscopic model with capacity at stations and along tracks is

considered. In the single tracks between stations A and B, between B and C, and between C and D, only one train at a time is allowed, while trains can overtake each other at the four stations. Moreover, there is a minimum time separation of 2 time units between consecutive trains at all stations and signals. Dwell time of trains at stations A, B and D is also equal to 2 time units.

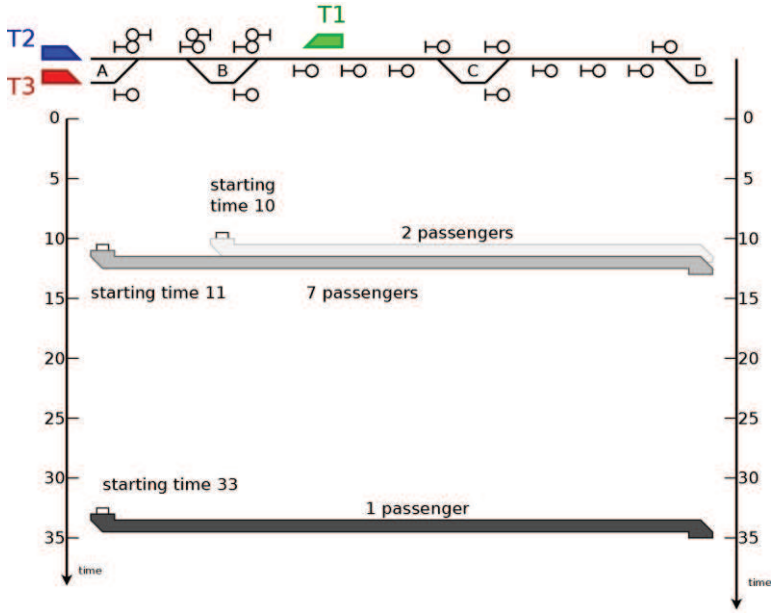

Figure 4: Infrastructure, trains and passenger demand of the illustrative example

In the timetable there are 3 trains (labelled T1, T2 and T3), with different line patterns: T1 goes from C to A, stopping in B and A; T2 goes from A to D, stopping in B; and T3 travels from A to D and arrives at station D just after T2. Finally, T3 is a fast train stopping at A and D only. At time h_0=0 train T1 is running from C to B, while T2 is departing from A. T3 starts its journey from station A as well, but later in time. The timetable is reported as a time-distance diagram in Figure 5(left). For each plot, time increases downwards on the vertical axis, while space is along the horizontal axis. In planned operations, trains T1 and T2 meet at station B, after which they move towards their respective destinations.

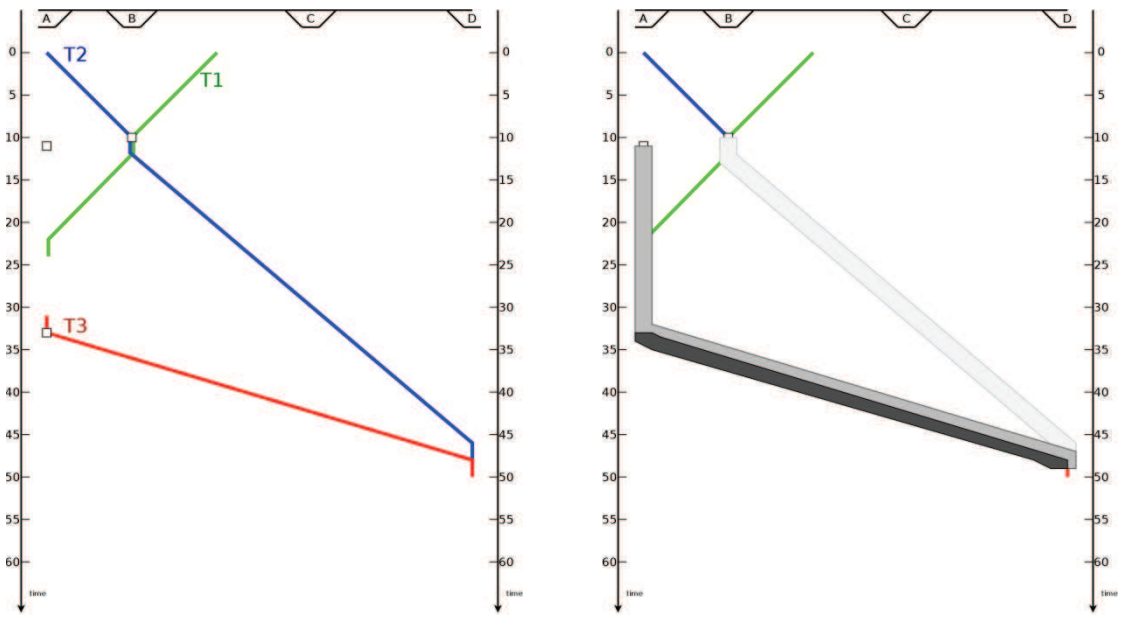

Figure 5: Planned timetable (left), and the corresponding passenger routing (right)

Figure 5(right) illustrates the assignment of passengers to the three trains, in case of planned operations. For each odw, a thick line is reported, distinguished by colour, that connects time and place of origin with time and place of destination, along the train services and stations used. Passengers in demands AD1 and AD2 take T3 to reach their destination, while passengers in BD1 take T2. In planned operations, the departure time of BD1 is 10; and the arrival time of BD1 is 46, for a travel time of 36 time units. For demand AD1 the departure time is equal to 11, and these passengers must wait for T3, thus having arrival time equal to 48, for a travel time of 37. For the last demand AD2, the departure time is 33, the arrival time is 48, and the travel time is 15. The total passenger travel time is thus $36 \times 2 + 37 \times 7 + 15 \times 1 = 346$. Assume that, in a real-time perturbed situation, the departure time of T2 is delayed by 10 time units. We next show the solutions provided by the three heuristics and the optimal solution of the MDM problem obtained by solving problem (A1)-(A11).

Figure 6 shows the solution provided by H1. According to H1, the train order remains fixed as in the timetable, while train departure times, connections plans, and passenger routing are optimized. In this case, T2 is delayed by one additional time unit in station B with respect to its delayed departure time 10, so that demand AD1 can use it. As a consequence, also the departure times of T1 from B and of T3 from A are delayed by one time unit. AD2 still uses T3, which suffers from a small delay on departure, and from an additional delay caused by T2. In the solution, the arrival time for BD1 is 57, for a travel time of 47. The arrival time of AD1 is 57, for a travel time of 46, while the arrival time of AD2 is 59, for a travel time of 26. The total travel time is thus 442.

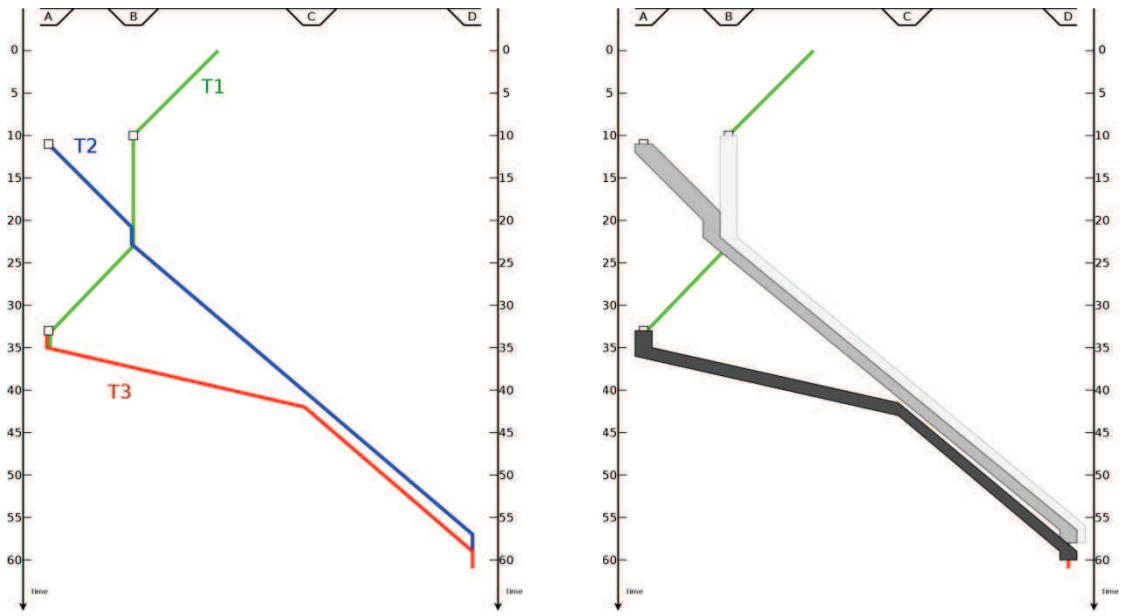

Figure 6 Solution provided by the H1 heuristic, train schedule (left) and passenger routing (right)

Figure 7 shows the solution provided by H2. According to H2, the timetable train sequence is kept, and the departure time of each train is delayed by the minimum amount necessary to achieve feasibility disregarding the passengers, as shown in Figure 7(left). The passenger routing obtained by solving problem (D1)-(D5) is shown in Figure 7(right). In this case, the (delayed) arrival time of BD1 is 56, for a travel time of 46 time units. Concerning OD pair A-D, since the departure time of T2 from A is 10 and the arrival time of passengers in AD1 is 11, these passengers must wait for train T3 and their arrival time is 58 (travel time = 47), the same as the arrival time of AD2 (travel time = 25). The total passenger travel time is thus 446.

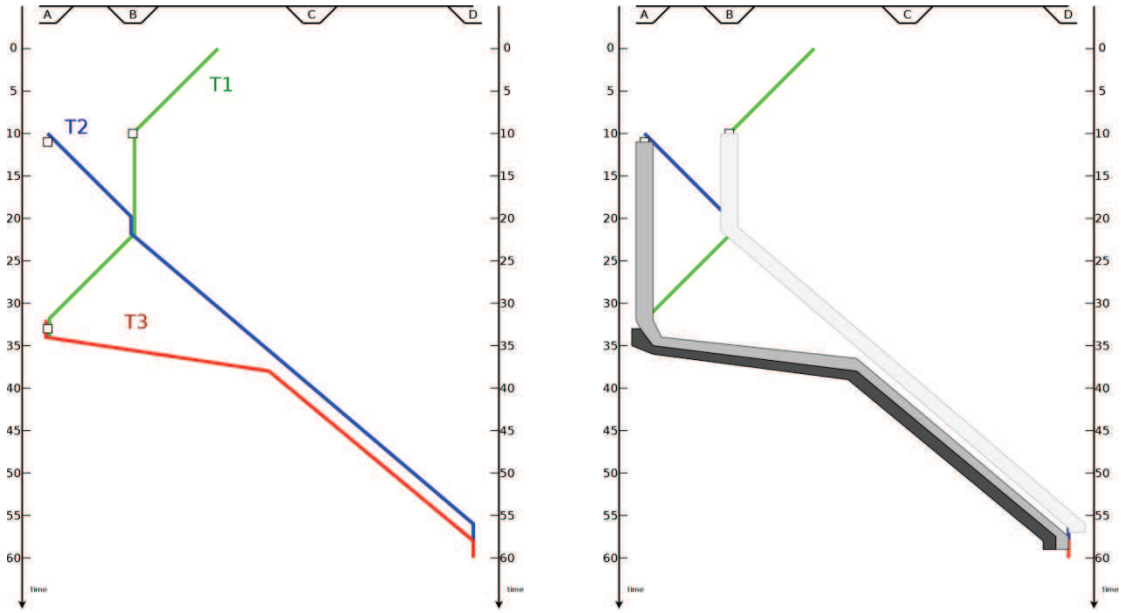

Figure 7: Solution provided by the H2 heuristic, train schedule (left) and passenger routing (right).

Figure 8 illustrates the solution found by H3 after the first iteration, i.e., when the problem (E1)-(E7) is solved starting from the solution found by H2 and then the problem (D1)-(D5) is solved consequently. The train rescheduling step finds beneficial to reorder T2 and T3 at station C, so that the passengers are rerouted through T3 in the subsequent solution of (D1)-(D5). The second iteration of H3 reschedules again the trains reaching the same schedule of Figure 9(left) and the subsequent passenger routing yields the optimal solution of Figure 9(right), which cannot be further improved.

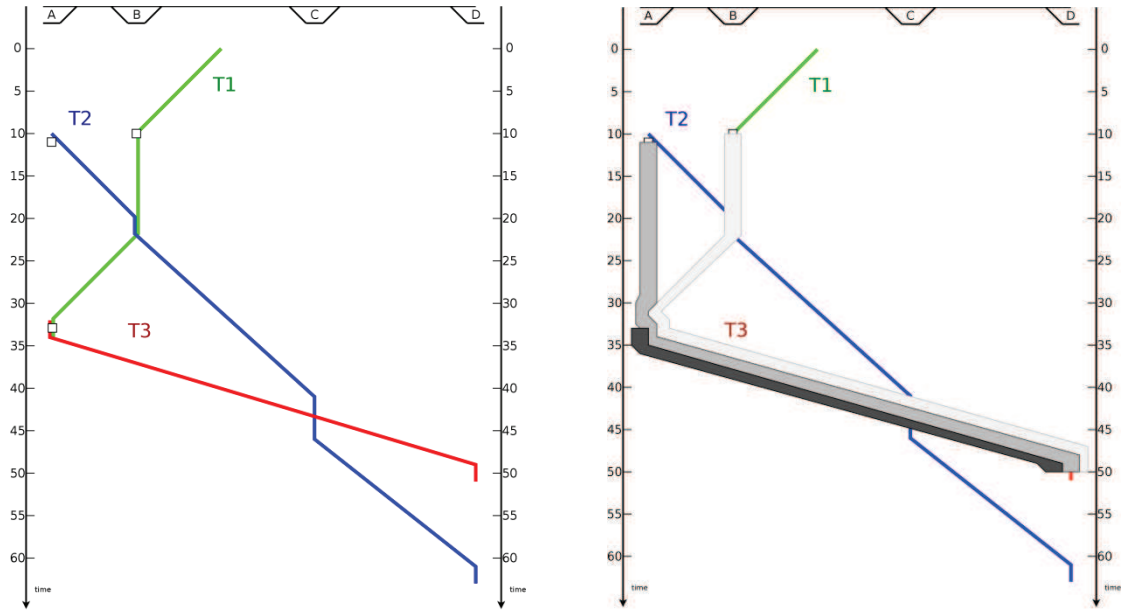

Figure 8: Solution provided by H3 after the first iteration, train schedule (left) and passenger routing (right).

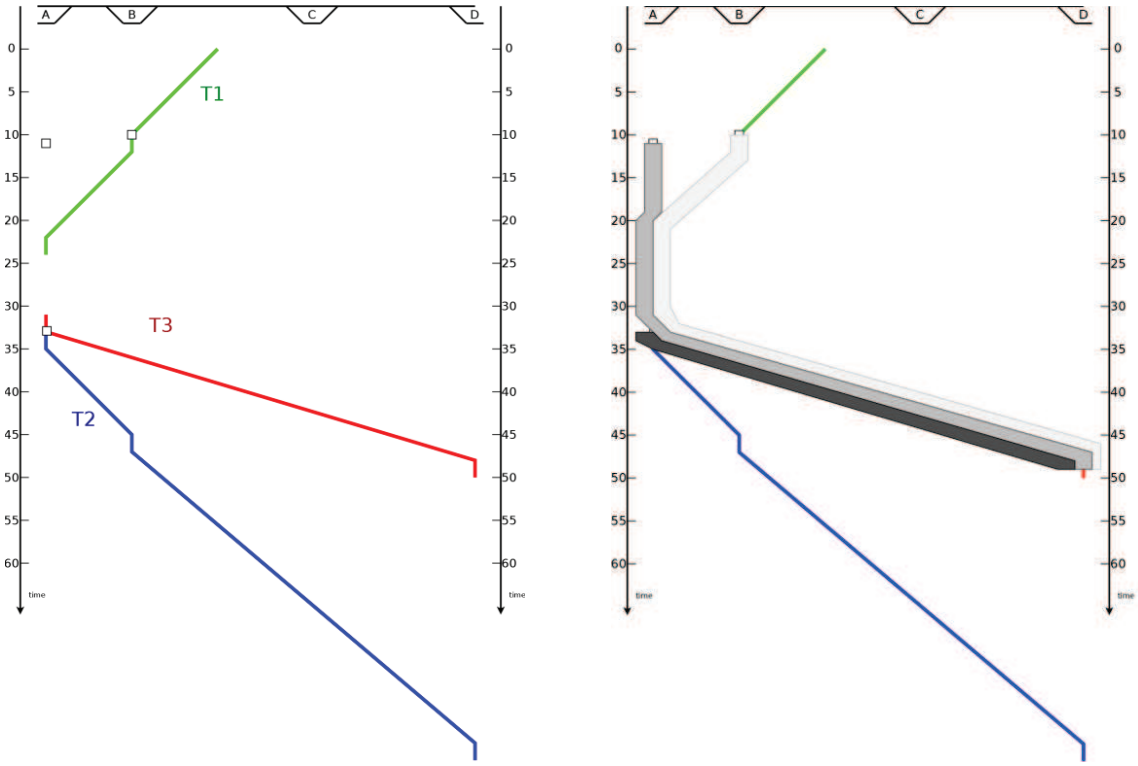

Figure 9: Optimal solution of the MDM problem, train schedule (left) and passenger routing (right).

Figure 9 shows the solution provided by H3 after the second iteration, which is also the optimal solution to the MDM problem (A1)-(A11). On the left the new train schedule is shown while on the right the passenger routing is reported. In this case, the solution prescribes a different train order in which T3 gets precedence over T2 also at station A. Besides T2, the other trains run unhindered. The passenger routing, shown in Figure 9(right), is the same as in the previous iteration, but the total passenger travel time slightly decreases due to the reordering of T2 and T3. In this solution, the arrival time for BD1 (via station A) is 48, for a travel time of 38; the arrival time AD1 is 48 for a travel time of 37; and the arrival time of AD2 is 48, for a travel time of 15. The total travel time is thus 350. Note that train T2 is severely delayed, but since no passenger takes it, this delay does not affect the objective function of the MDM problem.

Note that in the optimal solution passengers in BD1 take a longer route in terms of kilometres but with the smallest possible travel time. This routing flexibility is incorporated into the models (A1)-(A11), (C1)-(C10) and (D1)-(D5), i.e., for each demand, all routing options from origin to destination are included into these models.

8. Experimental assessment

This section reports on the results of a campaign of experiments on the performance evaluation of the three heuristics described in Section 6 with respect to the optimum of the problem (A1)-(A11) or to a lower bound on the optimum obtained by solving the problem (B1)-(B5) to optimality. All experiments are executed on a PC equipped with a processor Intel i5 CPU at 3.20 GHz, 8 GB memory. The commercial solver CPLEX 12.4 is used to solve the MILP.

8.1. Description of the instances

The instances studied in this paper are based on the Dutch railway network between Utrecht and Den Bosch, that has a line topology long about 40 km and comprises about two hundreds block sections. There are three major stations, Utrecht (Ut), Geldermalsen (Gdm), and Den Bosch (Ht). In all the instances we consider all trains stop

at those three stations. Furthermore, there are 8 minor stations, as detailed in Figure 10: Utrecht Lunetten, Houten, Houten Castellum, Culemborg, all located between Utrecht and Geldermalsen, and Zaltbommel between Geldermalsen and Den Bosch. Only local trains stop at all those minor stations. We consider trains running in both directions along the line, with the two traffic directions basically independent from each other. Trains can overtake each other at the outbound interlocking area in Utrecht, at the 4-tracks area around Houten (Htn), at Geldermalsen, and at the outbound interlocking area of Den Bosch.

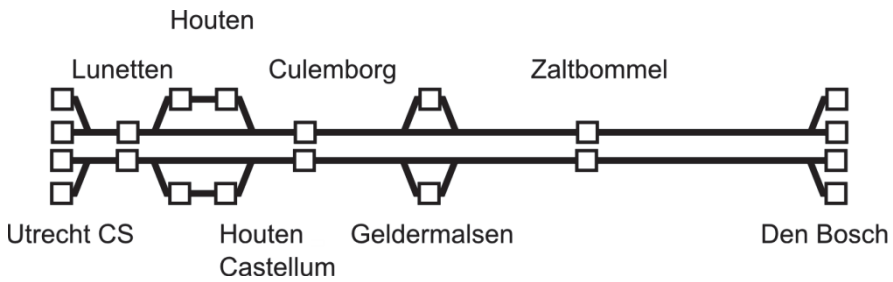

Figure 10. The railway infrastructure

The actual timetable used in operations in 2010 is considered, that is periodic with a period of 30 minutes. No freight trains are considered in the tested instances. For passenger trains, we consider three sets of instances, one with the same train frequency as in the actual timetable, and other two sets with lighter timetables, having a reduced number of trains. Namely, the actual timetable schedules 4 intercity trains per hour per direction between Utrecht and Geldermalsen, 4 local trains per hour per direction between Utrecht and Geldermalsen, and 2 local trains per hour per direction between Geldermalsen and Den Bosch. We refer to this as a timetable with train frequency 16 trains/hour, which is the amount of trains per hour in the busiest stretch of the network. This first set of instances use the same timetable described in Dollevoet et al. (2012, 2014A, 2014B) even if this paper considers the microscopic description of part of the network studied in those papers, between the stations of Utrecht and Den Bosch.

Lighter timetables are generated by incrementally removing two intercity trains per hour per direction (yielding a train frequency of 12 trains / hour) and removing the two local trains per hour per direction connecting Utrecht and Geldermalsen (train frequency of 8 trains / hour). The basic hourly pattern of the running trains is reported in Figure 11, for the three variants, in terms of time-distance diagrams. Time is reported on the vertical axis, distance on the horizontal axis. Intercity trains are depicted in blue; local trains in green. The dotted lines show the first trains of the next hour for each timetable.

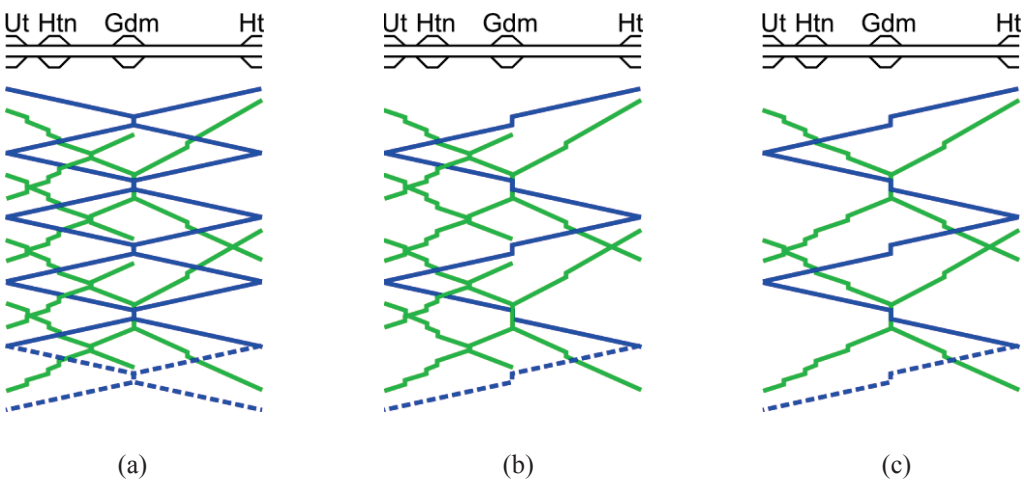

Figure 11. The three hourly timetables considered: 16 trains/ hour (a); 12 trains/hour (b); 8 trains/hour (c).

We next describe the OD pairs considered in the experiments. Due to the unavailability of detailed real data about passenger flows, we resort to realistic synthetic OD data based on the average flow of passengers at the considered stations as published by the infrastructure manager. Specifically, we first estimate the amount of passengers travelling on the line, and we then assign those flows of passengers proportionally to the stations considered, in proportion of the amount of passengers entering/exiting each station. This is translated to an average rate of passenger generation per OD, per time unit. Out of the theoretically 56 possible combinations of origin and destination stations, in a first set of instances we consider the 36 OD pairs with the largest amount of passenger flow, while in a second set of instances we limit the experiments to the 8 largest OD pairs.

Figure 12 gives a pictorial representation of the considered OD flows. The stations names are plotted along the circle from Utrecht (left) to Den Bosch (right). Each station has a colour (for instance red for Utrecht, towards green for Den Bosch). OD flows of passengers between pairs of stations are represented with a line connecting the stations, with the same colour of the departure station, and a thickness corresponding to the amount of passengers flowing on it. Based on these estimated OD flows, a demand *odw* is defined for each departure from the origin station, having a starting time π_{odw} equal to the scheduled departure time of the associated train. Given the values π_{odw}, and the passenger arrival frequency at station o, the number of passengers n_{odw} in each demand are chosen equal to the interval between consecutive train departures multiplied by the passenger arrival frequency at station o.

To reduce the instance complexity, we restrict the time horizon of traffic control to different lengths, in order to study the impact of an increasing amount of trains and more passengers travelling on the network. The time horizon is the time interval considered for optimization. In this section we study time horizons of 30, 45 and 60 minutes. For example, a time horizon of 45 minutes with the timetable scheduling 16 trains per hour corresponds to an instance with 12 trains, on average. In this instance, considering 8 OD pairs corresponds to having 39 demands *odw,* on average.

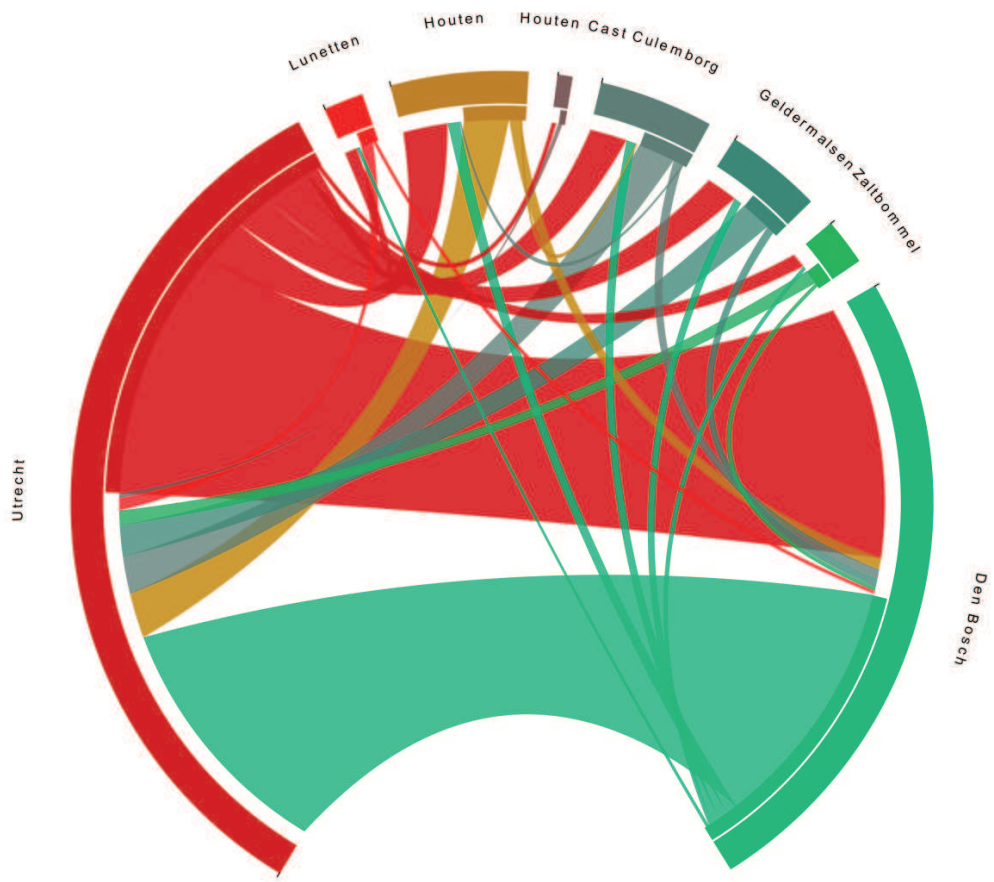

Figure 12. Graphical representation of passenger flow along 36 OD pairs, between the 8 stations considered.

Entrance delays, for all trains in the network, are defined based on a three-parameter Weibull distribution fitted to real data, computed as in Corman et al. (2011). In more detail, we generated different groups of instances by varying the train frequency, the number of OD pairs and the time horizon. In each group we consider 20 instances each with a different train delay pattern, generated according to a typical Monte-Carlo scheme. The first 10 instances represent normal traffic conditions, and are generated with the same Weibull distribution used in Corman et al. (2011) . The second 10 instances represent more perturbed traffic conditions, and are generated by using a Weibull distribution with the same scale and shift parameters of the first 10 instances and a doubled shape parameter. All computational results are reported as average over this combination of 10 + 10 instances (when not aggregated at higher level). Further study might investigate the existence and description of a functional or empirical relationship between the statistical description of entrance delays, and the gap in performance of passenger travel time.

8.2. Experiments with 8 OD pairs and up to 1 hour time horizon

We next report on the experiments for the instances with 8 OD pairs and a time horizon varying from 30 minutes to one hour. Table 2 shows the size of the considered instances, for the different combinations of train frequency and time horizon, in terms of the number of binary variables of three models, namely the MDM model (A1)-(A11), the train scheduling model TS (E1)-(E7) and the delay management model DM (D1)-(D5). The train frequencies considered are 8, 12, and 16 trains per hour (trh). The considered time horizons are 30, 45, 60 minutes.

MDM Model	30 min	45 min	60 min
8 trh	2137	5292	21192
12 trh	10497	21394	49613
16 trh	21923	42913	102048

TS+DM	30 min	45 min	60 min
8 trh	66+2071	205+5087	445+20747
12 trh	258+10239	535+20859	1325+48288
16 trh	445+21478	1024+41889	2192+99856

Table 2 Number of binary variables for the various approaches

Tables 3 and 4 report on the average values of the objective function and on the computation time of all approaches. We show the average travel time of each passenger instead of the total travel time of all passengers, to make the comparison easier when instances with different number of passengers are considered. Each line reports the average result over 20 instances with the same train frequency and time horizon. The following results are reported in the tables: the value found by H1 within 3 minutes or 1 hour of computation, the value found by H2, the value found by H3 and the values found by MDM within 3 minutes and 4 hours of computation. Column 3 of Table 2 shows the average best known upper bound for each group of instances. When H1 or MDM was not able to find a feasible solution for some instances, within the time limit of computation, we consider under H1 the solution found by H2, that is always computed within a few seconds.

Train Frequency	Time Horizon	Best Known Upper Bound	H1 (3 min)	H1 (1h)	H2	H3	MDM (3 min)	MDM (4h)
8trh	30 min	1729.4	**1729.4**	**1729.4**	1730.3	1730.7	**1729.4**	**1729.4**
8trh	45 min	1734.4	1736.0	1736.0	1755.2	1738.1	**1734.4**	**1734.4**
8trh	60 min	1654.9	1662.3	1661.0	1755.1	1747.1	1712.7	**1654.9**
12trh	30 min	***1544.6***	1557.3	1557.3	1570.8	1558.1	1545.3	**1544.8**
12trh	45 min	***1883.3***	1901.4	1899.0	1975.8	1969.4	1911.1	**1885.8**
12trh	60 min	1844.1	1855.4	1854.8	1855.4	**1844.1**	1950.8	1872.1
16trh	30 min	1684.0	1713.1	1712.7	1736.0	1711.1	1690.1	**1684.8**
16trh	45 min	***1734.4***	1756.0	1742.7	1768.9	1741.5	1953.4	**1741.4**
16trh	60 min	1706.4	1740.9	1739.9	1740.9	**1706.4**	1740.9	1740.9
Average		*1724.0*	*1739.1*	*1737.0*	*1765.4*	*1749.6*	*1774.2*	*1732.1*

Table 3 Value of the average passenger travel time [sec] obtained by the various approaches.

Some trends are clearly visible in the results of Table 3. For each group of instances, the approach achieving the best average result is highlighted in bold. In most cases, the approach in bold finds the best upper bound for all the 20 instances. In a few cases (highlighted in bold italic in the third column) the best upper bound for the different instances in the same group is found by different approaches. As expected, the best results are achieved by the MDM model, though after a huge computation time. However, for the largest instances with 12 or 16 trains/hour and 60 minutes of time horizon, the MDM model is not able to outperform H3 even after 4 hours of computation.

Train Frequency	Time Horizon	H1 (3 min)	H1 (1h)	H2	H3	MDM (3 min)	MDM (4h)
8trh	30 min	0.2	0.1	0.2	0.4	0.3	0.1
8trh	45 min	0.7	0.6	0.3	0.8	1.4	0.8
8trh	60 min	176.3	2063.4	0.7	2.1	180.0	3858.0
12trh	30 min	115.5	273.3	0.5	1.3	173.5	2264.0
12trh	45 min	180.0	1621.8	1.5	4.4	180.0	3129.6
12trh	60 min	180.0	3193.8	2.2	5.7	180.0	12273.0
16trh	30 min	180.0	2403.6	0.7	2.2	180.0	2323.4
16trh	45 min	180.0	3175.1	1.4	4.2	180.0	10434.8
16trh	60 min	180.0	3600.0	3.5	9.9	180.0	14150.2
Average		*132.5*	*1814*	*1.2*	*3.5*	*139.5*	*5381*

Table 4 Computation time [sec] for the approaches considered

Table 4 details the average computation time of the different approaches for the nine groups of 20 instances. When no feasible solution is found, the timetable solution is used, and the total time limit of computation is reported. H2 and H3 are the fastest heuristics, being able to deliver, on average, a solution within 10 seconds even for the largest instances. For the iterative approach H3, the average number of iterations is 1.9, ranging from 1.2 iterations for the smallest instances to 2.1 iterations for the largest instances. The amount of iterations for all instances never exceeds 3. From an overall point of view, H3 performs best with respect to the other heuristics, given that good solutions are computed within a few seconds, compatible with real time applications.

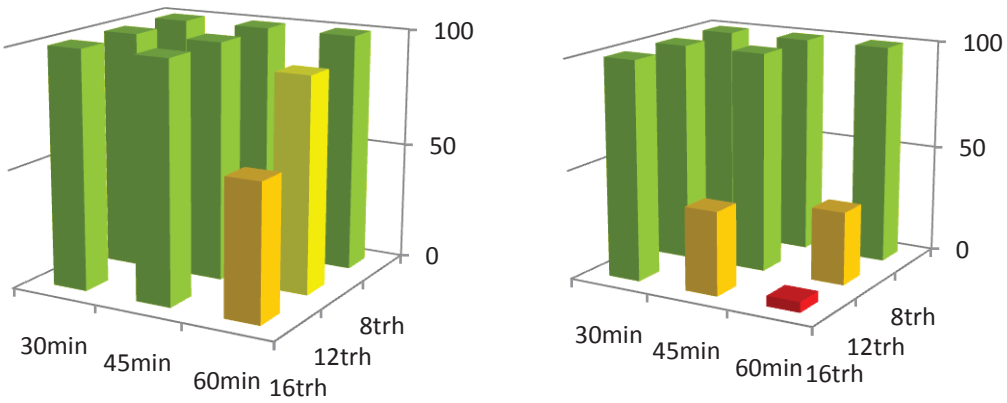

Figure 13. Percentage feasibility rate (left) and optimality rate (right) for MDM for the nine groups of instances.

Figure 13 illustrates the percentage of feasible (left) and optimal (right) solutions found by MDM within a time limit of 4 hours for each of the nine considered groups of instances. The colours are related to the following intervals: green =100%; yellow [66%, 100%), orange [33%, 66%); red [0%, 33%). The feasibility rate quickly decreases, for the instances of 60 minutes, as the number of trains per hour increases. Optimality is lost even earlier: only 4% of the largest instances can be solved to optimality within 4 hours of computation.

Table 5 reports on the average lower bound on the optimum for each group of instances, computed with the approaches described in Section 5. For the approaches of Sections 5.1 and 5.2 the computation time is also

reported. Column 3 shows the best known lower bound for each group of instances, obtained as the maximum among the values computed with the approaches of Section 5 and the optima (when known) or the lower bound to the MDM model after 4 hours of computation. As for Table 3, here we show the average travel time per passenger and each line is the average over the 20 instances of Tables 3 and 4. As shown in Table 5, the lower bound proposed in Section 5.2 is very good for the largest instances, since it allows to keep the computation time below 45 seconds, on average, while the lower bound quality deteriorates less than 0.6% with respect to the best known lower bound. On average, the deterioration over all groups of instances is below 6%.

Train Frequency	Time Horizon	Best Known Lower Bound	Lower Bound of Sec 5.1		Lower Bound of Sec 5.2	
			LB Value	Comp Time	LB Value	Comp Time
8trh	30 min	1729.4	1729.4	0.3	1687.9	3.4
8trh	45 min	1734.4	1734.4	1.3	1660.3	5.4
8trh	60 min	1655.4	1595.1	2963.8	1540.7	14.0
12trh	30 min	1544.8	1530.2	1319.0	1463.1	10.0
12trh	45 min	1887.2	1861.5	2068.9	1720.6	15.0
12trh	60 min	1779.9	1741.6	3173.3	1621.0	27.8
16trh	30 min	1684.8	1673.8	2016.3	1585.6	14.1
16trh	45 min	1718.2	1673.9	2876.3	1620.6	20.7
16trh	60 min	1575.3	1566.7	3600.0	1566.7	44.6
Average		*1701.1*	*1678.5*	*2002.1*	*1607.4*	*17.22*

Table 5 Lower bounds in terms of average passenger travel time [sec] and computation time [sec]

Table 6 illustrates the average optimality gap 100(*UB-LB*)/*LB* for all approaches, i.e., the upper bounds of Table 3 achieved by each approach are compared with the best lower bounds of Table 5. The MDM approach allows to find the proven optima for 6 out of 9 groups of instances. For those instances when MDM does not find the optimum, H3 provides the best solutions, on average. We observe that only H2 and H3 are compatible with real-time applications and that, between these approaches, H3 outperforms H2 on most of the instances. When comparing H3 with the best performing approach, we observe an increase of about 1% in the optimality gap, which is quite a small price to pay to achieve real-time performance.

Frequency	Time Horizon	H1 (3 min)	H1 (1h)	H2	H3	MDM (3min)	MDM (4h)
8trh	30 min	0.00	0.00	0.05	0.08	0.00	0.00
8trh	45 min	0.09	0.09	1.19	0.21	0.00	0.00
8trh	60 min	0.41	0.34	5.68	5.25	3.34	0.00
12trh	30 min	0.81	0.81	1.66	0.85	0.03	0.00
12trh	45 min	0.75	0.62	4.48	4.17	1.25	0.00
12trh	60 min	4.07	4.04	4.07	3.48	8.76	4.93
16trh	30 min	1.65	1.63	2.95	1.54	0.32	0.00
16trh	45 min	2.15	1.41	2.87	1.34	12.04	1.33
16trh	60 min	10.01	9.96	10.01	8.19	10.01	10.01
Average		*2.22*	*2.10*	*3.66*	*2.79*	*3.97*	*1.81*

Table 6 Optimality gap in percentage versus the best known lower bound

8.3. Experiments with 2 hour time horizon

This subsection presents our computational experience with significantly larger instances compared to the previous subsection, with two hours of time horizon and with a timetable of 16 trains/hour, i.e., the actual train frequency observed in real-life. Two sets of OD pairs are considered, namely the 8 OD pairs of the previous subsection and the full set of 36 OD pairs. Similarly to the previous subsection, 20 instances with 8 OD pairs and 20 instances with 36 OD pairs are generated with the distributions of Section 8.1, and the results are reported as average over each group of 20 instances.

Each instance with 8 OD pairs contains, on average, more than 12000 passengers grouped in 119 demands, while an instance with 36 OD pairs contains, on average, more than 14500 passengers grouped in 290 demands. The resulting MDM models contain more than 400 thousands variables for the instances with 8 OD pairs and more than 2 millions variables for the instances with 36 OD pairs. Due to the increased size of the instances, no feasible solution is found by CPLEX when solving the MDM or H1 MILP models, even after 8 hours of computation. Therefore, we next report on the performance of H2 and H3 only.

Table 7 gives the average passenger travel time and the computation time of H2 and H3, both in seconds. H3 converges after 2.5 iterations on average (5 iterations in the worst case). For the instances with 8 OD pairs, the optimality gap is 11% for H3 and 13.7% for H2. For the instances with 36 OD pairs, the optimality gap is 12.3% for H3 and 15.2% for H2. It is worth noting that the decrease in the average passenger travel time when passing from 8 to 36 OD pairs is due to the fact that the 24 additional OD pairs have a shorter travel time, on average, compared to the original 8 OD pairs.

	#OD pairs	H2	H3	Lower bound of Sec 5.2
Average Passenger Travel Time (sec)	8	1810.7	1767.1	1591.9
Computation time (sec)	8	18.9	68.7	11.1
Average Passenger Travel Time (sec)	36	1763.7	1719.2	1531.1
Computation time (sec)	36	63.6	221.7	73.7

Table 7 Average performance on the 20 two-hour traffic instances.

9. Conclusions and future research

This paper integrates train scheduling and delay management visions into an MDM model to control railway traffic in real-time with the objective of minimizing passenger travel time. Research on TS and DM in the recent years has been very active, even if this paper represents the first attempt to fully incorporate the passengers' point of view into a microscopic railway traffic rescheduling model.

The MDM problem is modelled as a MILP that can be solved to optimality by commercial solvers for small-size instances. A new lower bound provides good values within a short computation time also for the largest instances. Three heuristic procedures are developed, based on decomposing the problem into a train scheduling problem and a passenger routing problem. Two of the three heuristics are able to find near-optimal solutions also for large and complex cases, for which the MDM approach fails to compute a feasible solution. The time to compute lower and upper bounds is quite limited, and compatible with real-time applications. Overall, computational experiments on a real-life network with a large set of OD pairs show that H3 is promising to increase railway customer satisfaction in response to real-time perturbations.

There are several possible directions open for future research, starting from these results. More efficient approaches are needed to control large networks in real time, with a large set of OD pairs, both in terms of lower and upper bounds. The lower bound proposed in this paper might be used within a heuristic or exact approach for computing good solutions within a short computation time. The MDM model could be enriched to take into account the finite capacity of each train and/or more sophisticated measures of the passenger discomfort. For example, Meschini et al. (2007) and Sato et al. (2013) propose more careful models than the passenger travel time minimization. In principle, the MDM model can easily incorporate different discomfort measures by weighting differently the time spent by each passenger onboard the train or in the station, or even considering a discomfort increasing with the number of passengers travelling on the same train. Finally, one could also take into consideration the anxiety of the passengers, which increases with the risk of missing a connection as a train delay increases.

A further line of research concerns with the design of more sophisticated models for the passengers' behaviour: how are passengers going to react when a change to their preferred path is suggested? Are they going to stick to their (offline) decision or are they going to follow the real-time suggestion for alternative modes of connectivity? Approaches based on discrete choice theory might be helpful to model these questions. The analysis of recorded passenger flows might provide insights on how passenger flows react to unexpected events as investigated, for example, by van der Hurk et al. (2013).

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