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Scheduling models for optimal aircraft traffic control at busy airports: tardiness, priorities, equity and violations considerations

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ABSTRACT

This work addresses the real-time optimization of take-off and landing at a busy terminal control area in case of traffic congestion. These areas are becoming the bottleneck of the entire air traffic control system, in particular in the major European airports, where there is limited possibility to build new infrastructure. This problem is particularly challenging, in order to effectively optimize aircraft operations, since it is necessary to incorporate all safety rules in the optimization model and to take into account a number of performance indicators that are important to evaluate the quality of a plan of operations. Moreover, solutions must be produced in very limited computation time. In this paper, detailed job shop scheduling formulations are proposed to take into account safety regulations and various performance indicators. The aim of this work is to investigate single objective functions able to represent a reasonable balance between the performance indicators. To this aim, the best known solutions with respect to various objective functions are compared. The quality gaps between the solutions proposed by the different formulations solved are quantified via a commercial solver. Experiments are performed for the two major Italian airports, Milano Malpensa and Roma Fiumicino. Disturbances are generated by simulating various sets of random landing/take-off aircraft delays. A comprehensive computational analysis makes possible the selection of those solutions that are able to find the best compromise among the different indicators and, consequently, the investigation of the most representative objective functions.

Keywords: Efficient Landing/Take-Off Operations; Microscopic Air Traffic Control Models; Non-Dominated Aircraft Scheduling Solutions; Mixed Integer Programming.

1 Introduction

The ever growing demand of air transport is increasing the pressure on air traffic controllers, since air traffic in peak hours is getting closer to the capacity of the Terminal Control Area (TCA), at least in the major European airports where there is limited possibility of creating new infrastructure. Aviation authorities are thus seeking optimization methods to better use the available infrastructure [2, 9, 14, 20]. However, the development and implementation of effective optimization methods for such operational problems requires the consideration of a number of aspects that are rarely taken into account simultaneously in scheduling theory:

- The optimization model should be able to incorporate all detailed information that is compliant with TCA safety regulations, including information which is not relevant for the air traffic flow management in large networks with multiple airports, and is therefore neglected in macroscopic models [6, 10, 22]. In most of the macroscopic models, the characteristics of the airport infrastructure are drastically simplified and the flight paths are aggregated, so that *potential conflicts* between single aircraft may be not visible, at least at the level of runways, ground and air segments of the TCA. A potential conflict occurs whenever aircraft traversing the same resource do not respect the minimum required safety distance.

- The time available for developing a new schedule can be very limited, since a computerized scheduler should be able to promptly react to any change occurring in the TCA.

- To a large extent, air traffic control operations and related issues are still scheduled by human controllers, who develop feasible schedules based on their past experience and intuition without using any formally defined performance indicator. The lack of a generally recognized performance indicator to optimize places importance on the definition of the objective function. The quality of a schedule typically involves several performance indices reflecting the interests of the different actors involved in air traffic management, such as the aircraft punctuality, the utilization level of airport resources, the costs incurred by different airline companies in terms of delays, broken flight connections and energy consumption, and so on. All these indices should be taken into account in the schedule development phase.

This paper addresses the first item by adopting a detailed microscopic formulation, based on a job shop scheduling problem. The proposed modeling approach takes into account all the relevant TCA safety aspects. The second item suggests that optimization models with a single objective function, possibly combining multiple indices, are more suitable than multi-objective approaches, since more efficient tools are available to solve these problems. This is also the most common choice in the literature (see, e.g., the reviews in [4, 7, 11, 21, 22, 26]). The present paper thus investigates single objective functions able to find a good compromise among the different indices listed in the third item. Specifically, we observe that aircraft typically fly at constant speed in the TCA and that at constant speed the energy consumption is almost proportional to the flying time. Therefore, we use the aircraft flow time as a surrogate of the energy consumption. Also, we use the minimization of the maximum completion time as a common surrogate for the throughput maximization. Moreover, we implicitly take into account the minimization of broken flight connections by minimizing the number of aircraft delayed more than a given threshold. So doing, we can translate all performance indicators in terms of aircraft arrival times at the entrance in the TCA and at the runways.

The Aircraft Scheduling Problem (ASP), we deal with in this paper, can be summarized as follows. Given a set of landing/take-off aircraft and for each aircraft its path in the TCA, its current position, its scheduled runway occupancy time and the required time window to accomplish the landing/departing procedures, the ASP is to assign the start time to each aircraft in all the resources it crosses in its path in such a way that all the potential conflict situations between aircraft are solved and a suitable objective function is minimized.

This work follows the approach of Bianco et al. [8], based on the no-wait version of the job shop scheduling problem, to build a microscopic formulation of the ASP in the TCA. However, we use the alternative graph model of Mascis and Pacciarelli [23] in order to increase the level of detail of the practical ASP model. Specifically, we include in the model relevant TCA aspects such as waiting in flight before landing, intervals for aircraft speeds, single capacity (named no-store or blocking [18]) constraints at runways, multiple capacity constraints of air segments, and so on. As for the objective function, alternative graph formulations of the ASP have been recently proposed in [13, 14, 15, 27] as a makespan minimization problem.

A set of ASP formulations are developed and tested for relevant objective functions. The focus of the computational study is on the assessment of solution quality. The ASP solutions are analyzed from the viewpoint of the above described performance indicators. The computational experiments have been carried out on practical size instances of the two main Italian airports in terms of passenger flows: Roma Fiumicino (FCO) and Milano Malpensa (MXP). For each airport, 40 delayed scenarios are considered and the resulting problems are solved with a commercial solver.

Section 2 will review the literature most relevant for this work. Section 3 will formally describe the modelling of specific ASP constraints. Section 4 will present the mathematical formulations. Section 5 will report the experiments conducted on the FCO and MXP instances. Section 6 will summarizes the paper results and outlines future research directions. Two appendix sections will finally illustrate the alternative graph formulation of a numerical example.

2 Literature review

This section briefly reviews recent papers on the general Air Traffic Flow Management (ATFM) problem. A more thoughtful discussion of the existing literature can be found e.g. in [4, 7, 11, 21, 22, 26]. The ATFM problem can be divided into two basic categories: the traffic control between airports (see e.g. [6, 9, 10]) and the traffic control in the TCA of a single airport (see e.g. [8, 13, 20]). For the former category, macroscopic models with multiple airports and aggregated flight paths are often adopted. For the latter category, microscopic models are proposed with identification and resolution of potential aircraft conflicts at the level of ground and air resources.

The algorithmic approaches are of various types. Among the heuristic approaches, fast heuristics are proposed in [14, 15, 16, 25], while exact procedures can be found in [13, 15, 16, 17].

Another classification is based on the type of information. When dealing with static information (see e.g. [13, 15, 16, 24]), the position and speed of all aircraft is known in the traffic prediction. The case with dynamic information (see e.g. [5, 19, 27, 30]) requires the computation of an aircraft schedule every time a new incoming aircraft is known.

In this work we focus on the development of microscopic TCA models in the case of complete information. The ASP problem is formulated as a linear mathematical program and solved via a commercial solver, trying to compute the optimal solution for a given performance indicator.

We now present a detailed review of some literature most related to our work, with specific reference to the different choice of the objective functions and constraints in the formulations.

Allahverdi et al. [1] and Ball et al. [3] show in their papers a detailed panoramic of different solving approaches used in the air traffic literature. In particular, the main objective functions discussed are the minimization of delays and costs. In fact, the costs are often calculated based on the deviation from the nominal schedule, i.e., in terms of aircraft delays.

Hu et al. [19] minimize the sum of the difference between the predicted and the allocated landing times of each aircraft. Eun et al. [17] try to limit the aircraft delays and the deviation from the estimated arrival time, by taking into account airline preferences. Alternatively, Ernst et al. [16] measure the cost associated with the deviation from the preferred aircraft landing time.

Beasley et al. [5] define an aircraft displacement problem and consider the cost of a solution adjustment procedure where, if an aircraft is further delayed with respect to an initial solution, an additional penalty has to be paid. Sölveling et al. [29], instead, include in the cost function the environmental impact, in terms of fuel and CO_2 emissions, when there are deviations from the nominal schedule. Soomer and Franx [28] combine cost functions declared by the airlines for each aircraft typology but, to avoid manipulation, rescale them, making the resulting solution fair.

Other works aim to maximize the use of airport capacity (i.e., the throughput). Bianco et al. [8] adopt the minimization of the maximum completion time for the throughput maximization, as previously described in Psaraftis [24].

The previous work dealing with the alternative graph formulation of the ASP described in [13, 14, 15, 27] was focused on the minimization of the maximum delay due to potential conflicts between aircraft traveling in a busy TCA during a time horizon of traffic prediction starting from a given reference time and lasting up to some hours. This formulation corresponds to a job shop scheduling problem with makespan minimization.

This work evaluates all the above proposed indices and other relevant performance indicators, with special emphasis on extent of the impact of each specific indicator on the others.

3 Problem description

In the TCA, landing aircraft move from an air entry point of the TCA to a runway via landing air segments, following a standard descent profile, while maintaining a minimum safety distance between every pair of consecutive aircraft, depending on their type and position (at the same or different altitude). Similarly, departing (take-off) aircraft leave the runway flying toward the assigned exit point via departing air segments along a standard ascent profile, still respecting the minimum safety distance. The space distance can be translated into a time distance, *setup time*, by taking into account the different aircraft speeds. Setup times are considered sequence-dependent, since the minimum distance between different aircraft categories (heavy, medium, light and others) depends on the relative order of processing of the common resources. For instance, the distance between heavy and light aircraft is much larger when light aircraft follow heavy aircraft than vice versa. Setup times do not only depend on the aircraft times but also on the route chosen for each aircraft.

Each aircraft has an assigned entry time into the TCA, which is the minimum time, *release time*, the landing/take-off procedure can start according to the current aircraft position and speed. Each aircraft has also scheduled times, *due date times*, to start processing some TCA resources.

The runway is a blocking (no-store) resource since it can only be occupied by one aircraft at a time. Each aircraft has a *processing time* on a runway and on the air segments before or after it, according to its landing/take-off profile. On the air segments, the processing time varies between a pre-defined time window, due to a limited possibility of aircraft speed changes.

Once an aircraft enters the TCA, it should proceed to the runway. However, before entering the airport area, *airborne holding time* can be used to make aircraft waiting in flight until they can be guided through their landing procedure, that means flying in circle in specific areas named *holding circles*. Once entered a holding circle, the aircraft must fly at a fixed speed for a number of half circles, as prescribed by the air traffic controller. Departing aircraft instead can be delayed in entering the TCA at ground level, i.e. before entering the runway.

A departing aircraft is supposed to take-off within its assigned time window and is late whenever it is not able to accomplish the departing procedure within its assigned time window. Following the procedure commonly adopted by air traffic controllers, we consider a time window for take-off between 5 minutes before and 10 minutes after the *Scheduled Take-off Time* (STT). A departing aircraft is considered delayed in exiting the TCA if leaving the runway after 10 minutes from its STT. Arriving aircraft are late if landing after their *Scheduled Landing Time* (SLT).

We use the following notation for the aircraft delays. *Entrance delay* is the delay of an aircraft at the entrance in the TCA. *Total exit delay* is the delay of an aircraft at the runway. The latter value is partly a consequence of a possible late entrance, which causes an *unavoidable delay* at the runway, and partly due to additional delays caused by the resolution of potential aircraft conflicts in the TCA, which is named *consecutive delay* [12, 13, 14]. A landing aircraft can have a consecutive delay at the entrance, if it is delayed in entering the TCA due to other aircraft scheduled on its entrance landing air segment. Landing and take-off aircraft can have a consecutive delay on a runway, if they have to give precedence to other aircraft in one or more TCA resources. These consecutive delays will be used to formulate the objective functions considered in this work.

3.1 Performance indicators

The ASP solutions can be many and can take into account different view points (airline companies, local and global authorities). In order to compare different indicators, we choose a set of objective functions, each one interested in looking at a particular aspect of the problem.

When minimizing the *aircraft tardiness*, we take into account *equity* that can be viewed as the computation of an ASP solution that minimizes the maximum consecutive delay. Also, a more global vision on the aircraft delays is considered that is the minimization of the average consecutive delay. In our approach, *aircraft priorities* are also taken into account as follows: landing aircraft, due to security measures, have greater priority than departing aircraft, which can wait at ground level with fewer risks. A landing, delayed aircraft has even greater priority, due to a lower level of fuel. In fact, giving priority to late aircraft (first-scheduled first-served) is a common concept in air traffic management, also referred as *fairness* [4]. So, the set of aircraft is divided in four classes which are ordered below in importance (from the highest to the lowest priority): 1) landing, delayed aircraft; 2) landing aircraft on time; 3) departing, delayed aircraft; 4) departing aircraft on time. Mirroring the two different approaches used to minimize delays, in this priority scenario we compare the solutions that take into account equity, defined here as the minimization of the average difference between maximum and minimum tardiness for each class, with the ASP solutions that minimize a weighted average tardiness with weights assigned to each aircraft according to the corresponding level of priority.

Another aspect to be taken into account when solving the ASP is the use of existing, critical airport resources. This consideration translates into the maximization of the throughput, that can be viewed as the minimization of the maximum completion time (see e.g. [8]). In our view, it corresponds to the landing/take-off time of the last aircraft traveling in the TCA during the time horizon (time span) of the traffic prediction. This objective function is compared with the minimization of the average completion time.

Finally, we evaluate the number of delayed aircraft exceeding a given tolerance thresholds and minimize the number of *deadline violations*, that imply additional operational costs for the airline companies, due e.g. to broken flight connections with other aircraft at the same TCA.

3.2 Terminal control areas

Figure 1 shows the TCA scheme of Malpensa airport. There are two runways (RWY 35L, RWY 35R), used both for departing and landing procedures. The MXP resources are three airborne holding circles (resources 1-3 in Figure 1, named TOR, MBR, SRN), eleven air segments for landing procedures (resources 4-14), a common glide path (resource 15), two runways (resources 16-17) and three departing air segments (resources 18-20, named SRN, TELVA, RMG). The common glide path resource includes two parallel air segments before the runways for which traffic regulations impose a minimum diagonal distance between aircraft added to a minimum longitudinal one.

Figure 2 presents the scheme of another TCA, Fiumicino airport. In this case, three runways (RWY 16L, RWY 16R, RWY25) can be used for departing and landing procedures, but two of them (RWY 16R and RWY 25) cannot be used simultaneously and are thus considered as one. The FCO resources are three airborne holding circles (resources 1-3 in Figure 2, named CIA, CMP, TAQ), seven air segments for landing procedures (resources 4-10), two runways (resources 12-13), a common glide path (resource 11) and three departing air segments (resources 14-16, named BOL, RAVAL, ELIVIN).

In this work, we assume that all take-off operations take place at a single runway (see

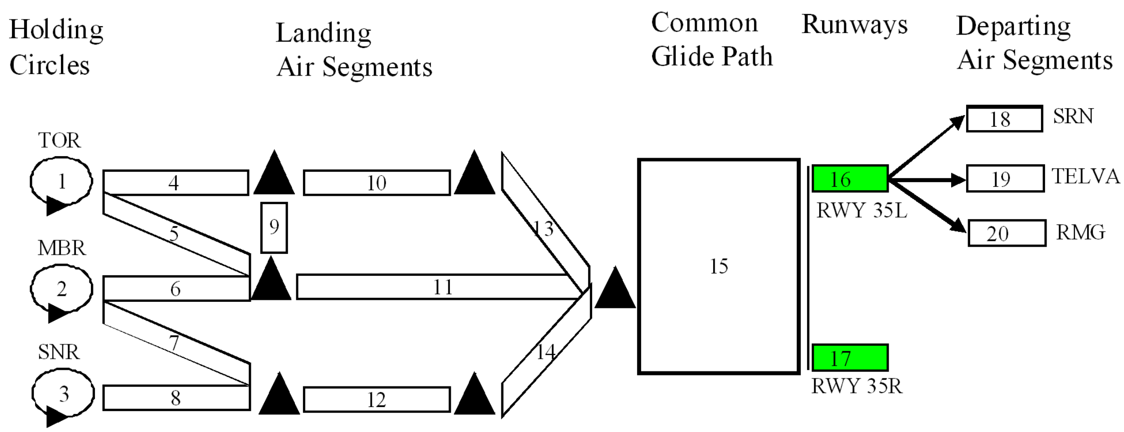

Figure 1: Malpensa (MXP) Terminal Control Area

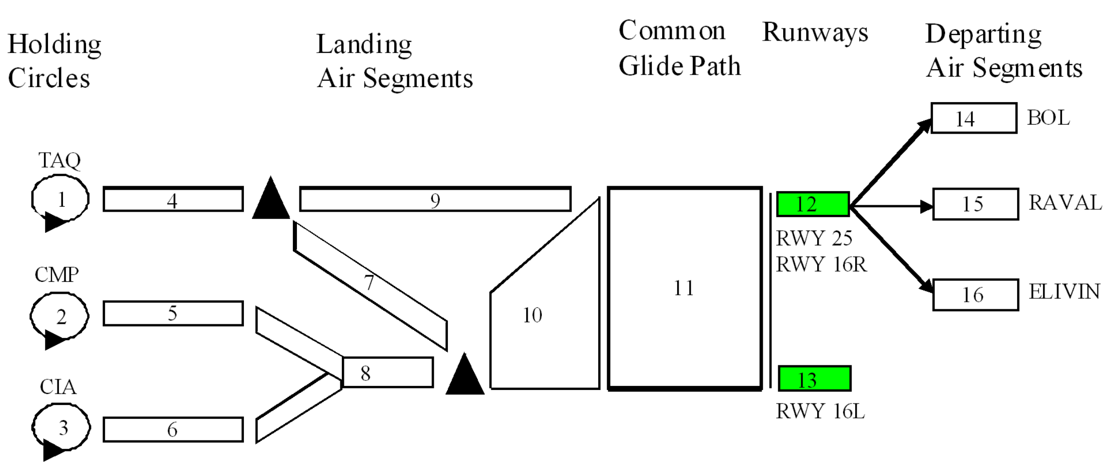

Figure 2: Fiumicino (FCO) Terminal Control Area

arcs leaving runway resource 16 for Figure 1 and runway resource 12 for Figure 2). We do not perform rerouting measures for departing and landing aircraft. D'Ariano et al. [14] focus on the combined aircraft reordering and rerouting problem at busy TCAs.

4 Problem formulation

In the general job shop scheduling formulation of the ASP, an *operation* denotes the traversal of a resource (i.e. air segment, common glide path, runway, holding circle) by a job (i.e. aircraft). The sequence of operations related to an aircraft represents the (predefined) route associated with that aircraft. The variables of the ASP are the start time t_i of each operation i to be performed by an aircraft on a specific resource. A set of route timings is *conflict-free* if, for each pair of operations associated to the same resource, the minimum time separation constraints are satisfied.

The ASP is represented by the alternative graph approach [23], since this model permits an accurate and efficient representation of the ASP [13, 14, 27]. Let $G = (N, F, A)$ be the graph composed of the following sets: $N = \{0, 1, ..., n\}$ is the set of *nodes*, where nodes 0 and n represent the start and the end operations of the schedule, while the other nodes are related to the start of the other operations; F is the set of *fixed directed arcs* that model the sequence of operations to be executed by an aircraft; A is the set of *alternative pairs* that model the sequencing decision. Each pair is composed of two alternative directed arcs.

Each node $i \in N$ of the graph is associated to the start time t_i of operation i, and corresponds to the entrance of the associated aircraft in the associated resource. By definition, the start time of the schedule is a known value, e.g. $t_0 = 0$, and the end time of the schedule is a variable t_n.

Each fixed directed arc $(i, j) \in F$ has a length w^F_{ij}, which is unique given i and j. The fixed arc length w^F_{ij} models a minimum processing time between the start of i and the start of j, such that $t_j \geq t_i + w^F_{ij}$. In particular, $\sigma(i)$ denotes the operation following i on its route. It follows that $(i, \sigma(i)) \in F$ is the directed fixed arc connecting i with $\sigma(i)$ and $t_{\sigma(i)} \geq t_i + w^F_{i\sigma(i)}$.

Each alternative pair $((i, j), (h, k)) \in A$ has two arcs with length w^A_{ij} and w^A_{hk}. The alternative arc length w^A_{ij} represents a minimum separation time between the start of i and the start j. In particular, w^A_{ij} is sequence-dependent, since it depends on the job related to node i and on the job related to node j. Also, there can be multiple alternative arcs between nodes i and j.

A *selection* S is a set of alternative arcs, at most one from each pair. An ASP solution is a *complete* selection S, where an arc for each alternative pair of the set A is selected, in which the connected graph *(N, F, S)* has no positive length cycles. Note that a positive length cycle represents an operation preceding itself, which is an *infeasibility*. Given a feasible schedule S, a timing t_i for operation i is the length of a longest path from 0 to i ($l^S(0, i)$). When minimizing makespan, an arc (k, n) between the end node k of each job and node n is added to the alternative graph, and a selection S is optimal if $l^S(0, n)$ is minimum over all the solutions.

The alternative graph can be viewed as a particular *disjunctive program*. We let X be the set:

$$X = \left\{ t \geq 0, x \in \{0,1\}^{|A|} : \begin{array}{ll} t_{\sigma(i)} - t_i \geq w^F_{i\sigma(i)} & \forall (i,\sigma(i)) \in F \ \ with \ \ \sigma(i) \neq n \\ t_j - t_i \geq w^A_{ij} - M(1 - x_{ij}) & \forall ((i,j),(h,k)) \in A \\ t_k - t_h \geq w^A_{hk} - M x_{ij} & \end{array} \right\} \tag{1}$$

The variables of the ASP are the following: $|N|$ real variables t_i associated to the start time of each operation $i \in N$ and $|A|$ binary variables x_{ij} associated to each alternative pair $((i,j),(h,k)) \in A$. The variable x_{ij} is 1 if $(i,j) \in S$, and $x_{ij} = 0$ if $(h,k) \in S$. The constant M is a sufficiently large number, e.g. the sum of all arc lengths.

The next subsections describe how the different types of TCA resources are modelled via alternative graphs, and show how each specific objective function can be formulated. A numerical example of a traffic situation is illustrated for the FCO airport. For the proposed ASP example, we give the trade-off between a set of non-dominated solutions, each one computed by solving a specific objective function to optimality. The graphs of the ASP example are reported in appendix.

4.1 Resources in the alternative graph model

The TCA is composed of various types of resources. This section illustrates how each of them is modelled in the alternative graph. Fixed arcs are depicted with solid arrows and alternative arcs are depicted with dotted arrows.

Figure 3(a) illustrates the formulation of a holding circle resource. We recall that holding circles are used by traffic controllers to let arriving aircraft wait before the start of their landing procedure when the TCA is congested. Let i be the entrance of aircraft A in the holding circle and let $\sigma(i)$ be the start of the next operation, the holding circle resource is formulated by two fixed arcs $(i, \sigma(i))$ and $(\sigma(i), i)$ (the two solid arrows), respectively of length $w^F_{i\sigma(i)} = 0$ and $w^F_{\sigma(i)i} = -\delta$, where δ is the time required to perform a holding circle, plus a pair of alternative arcs $((i, \sigma(i)), (\sigma(i), i))$ (the two dotted arrows) respectively of length $w^A_{i\sigma(i)} = \delta$ and $w^A_{\sigma(i)i} = 0$. The formulation of multiple (half) circles can be easily done in a similar way.

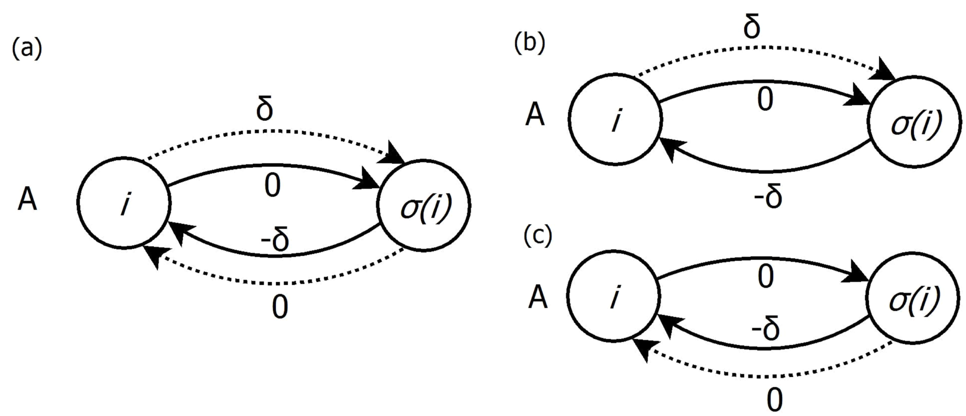

Figure 3: Alternative graph formulation of a holding circle (a), selected (b) and not selected (c)

Figure 3(b) shows the decision to perform a holding circle (the dotted arrow $(i, \sigma(i))$

is selected), while Figure 3(c) the case with no holding circle (the dotted arrow $(\sigma(i), i)$ is selected). The formulation of the holding circle constraints and variables follows:

$$\begin{aligned} &t_i - t_{\sigma(i)} \geq w^F_{\sigma(i)i} \\ &t_{\sigma(i)} - t_i \geq w^F_{i\sigma(i)} \\ &t_{\sigma(i)} - t_i \geq w^A_{i\sigma(i)} - M(1 - x_{i\sigma(i)}) \\ &t_i - t_{\sigma(i)} \geq w^A_{\sigma(i)i} - M x_{i\sigma(i)} \end{aligned} \tag{2}$$

If $x_{i\sigma(i)} = 0$ then no holding circle is performed, as shown in Figure 3(c).

Figure 4(a) presents the formulation of an air segment. We define operation i ($\sigma(i)$) as the entrance (exit) of aircraft A in the air segment. The travel time of aircraft A in the air segment must be included in a range $[w_{min}, w_{max}]$. To model this range of values two fixed arcs $(i, \sigma(i))$ and $(\sigma(i), i)$ are used with length $w^F_{i\sigma(i)} = w_{min}$ and $w^F_{\sigma(i)i} = -w_{max}$, respectively. The following constraints are thus required for the travel time of aircraft A:

$$\begin{aligned} &t_{\sigma(i)} - t_i \geq w_{min} \\ &t_i - t_{\sigma(i)} \geq -w_{max} \end{aligned} \tag{3}$$

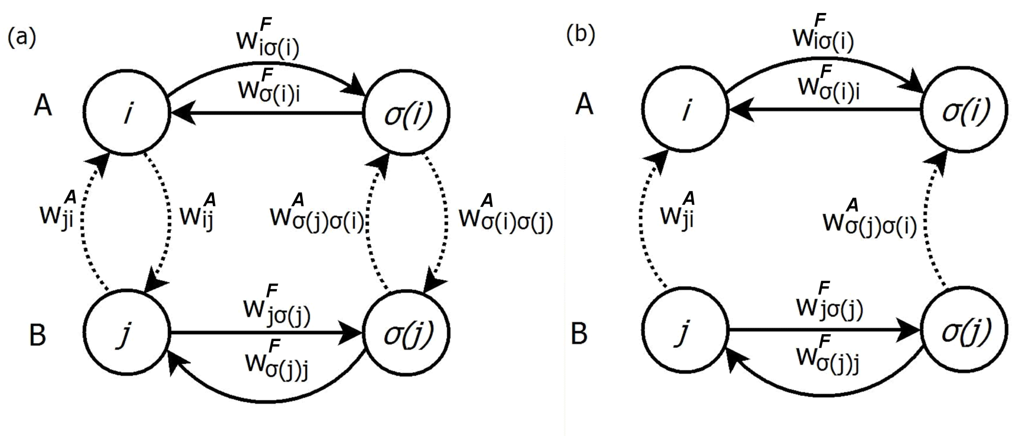

Figure 4: Alternative graph of an air segment (a), and a feasible arc selection (b)

In each landing/departing air segment, air traffic regulations impose a minimal longitudinal and diagonal separation distance between consecutive aircraft, that varies according to the aircraft category. The minimum separation time between two aircraft (named A and B in Figure 4) in the same air segment is thus formulated as a sequence-dependent setup time.

Since an overtake between two aircraft in the same air segment is not allowed for safety reasons, the entrance and exit orders between aircraft A and B must be the same. Thus, the sequencing decision variables are the two pairs of alternative arcs in Figure 4(a): $((i, j), (\sigma(j), \sigma(i))$ with lengths w^A_{ij} and $w^A_{\sigma(j)\sigma(i)}$; $((j, i), (\sigma(i), \sigma(j))$ with lengths w^A_{ji} and $w^A_{\sigma(i)\sigma(j)}$. The length of the alternative arcs is the sequence-dependent setup time between aircraft A and B.

In a feasible schedule (i.e. a complete selection with no positive length cycles), the selection of the two alternative arcs (i, j) and $(\sigma(i), \sigma(j))$ sequences aircraft A before B, i.e. the two constraints $t_j - t_i \geq w^A_{ij}$ and $t_{\sigma(j)} - t_{\sigma(i)} \geq w^A_{\sigma(i)\sigma(j)}$ are inserted in the graph.

Otherwise, the two alternative arcs (j, i) and $(\sigma(j), \sigma(i))$ must be selected (aircraft B before A).

To summarize, the formulation of the air segment sequencing decisions is next shown:

$$
\begin{array}{l}
t_j - t_i \geq w^A_{ij} - M(1 - x_{ij}) \\
t_{\sigma(i)} - t_{\sigma(j)} \geq w^A_{\sigma(j)\sigma(i)} - Mx_{ij} \\
t_{\sigma(j)} - t_{\sigma(i)} \geq w^A_{\sigma(i)\sigma(j)} - M(1 - x_{\sigma(i)\sigma(j)}) \\
t_i - t_j \geq w^A_{ji} - Mx_{\sigma(i)\sigma(j)}
\end{array}
\tag{4}
$$

If $x_{ij} = 0$ then the alternative arc of length $w^A_{\sigma(j)\sigma(i)}$ is selected, i.e. aircraft B is scheduled first in the air segment (as shown in Figure 4(b)). Consequently, the variable x_{ij} must be set to 0 (i.e. the alternative arc of length w^A_{ji} is selected); otherwise the alternative arc of length $w^A_{\sigma(i)\sigma(j)}$ is selected and a positive length cycle between nodes $\sigma(i)$ and $\sigma(j)$ is generated in the graph.

Figure 5(a) shows the runway formulation. Since there must be at most an aircraft at a time on each runway, a no-store resource is required. Let i, j be the entrance of aircraft A, B in the same runway and $\sigma(i)$, $\sigma(j)$ their exit. The alternative pair $((\sigma(i), j), (\sigma(j), i))$ is used to model the two possible sequencing decisions. For example, if aircraft A is scheduled first than aircraft B than the alternative arc $(\sigma(i), j)$ must be selected (see Figure 5(b)). The length of each alternative arc represents the setup time before on the runway. The formulation of this no-store resource is:

$$
\begin{array}{l}
t_j - t_{\sigma(i)} \geq w^A_{\sigma(i)j} - M(1 - x_{\sigma(i)j}) \\
t_i - t_{\sigma(j)} \geq w^A_{\sigma(j)i} - Mx_{\sigma(i)j}
\end{array}
\tag{5}
$$

If $x_{\sigma(i)j} = 1$ then aircraft A is scheduled first in the runway (Figure 5(b)).

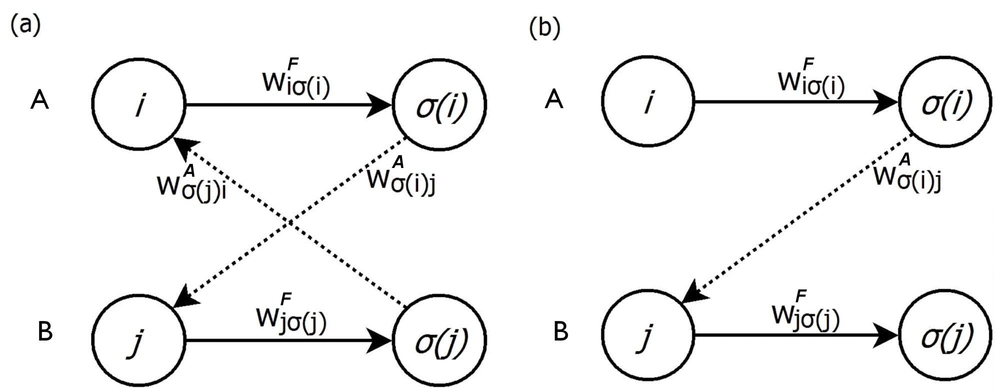

Figure 5: Alternative graph of a runway (a), and a possible arc selection (b)

Figure 6 presents a formulation of release and due date times. Aircraft A has only one operation in the TCA, that is modelled by node i. The release time r_i is the minimum time at which aircraft A can enter the TCA, modelled by the *release arc* $(0, i)$ between nodes 0 and i of length $w^F_{0i} = r_i$, i.e. $t_i \geq t_0 + w^F_{0i}$. The due date time d_i is the scheduled time of aircraft A, modelled by the *due date arc* (i, n) between nodes i and n of length $w^F_{in} = -d_i$, i.e. $t_n \geq t_i + w^F_{in}$. When an aircraft performs multiple operations in the TCA, its delay can be measured at any node i by means of a due date arc (i, n). The next

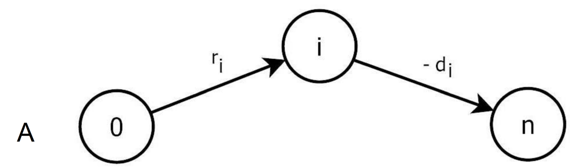

Figure 6: Release and due date arcs for aircraft A with one operation in the TCA

section will describe the formulation of delay minimization at the first node of the job and at a node of the job related to a runway resource.

4.2 Formulations with different objective functions

This section presents the alternative graph formulations of the ASP used in this paper. For each formulation, we consider all problem constraints introduced in the previous section and we describe how the specific objective functions have been modelled.

The first group of formulations is related to aircraft delay minimization. Here, we use two types of due date arcs: *entrance due date arcs* are associated with the first operation of landing aircraft and measure their entrance delay in the TCA; the *exit due date arcs* are associated with the operation modelling the entrance/exit in/from a runway resource of landing/departing aircraft and measure the delay caused by the resolution of potential aircraft conflict in the TCA. Both types of due date arcs measure the delay generated by the scheduling decisions, i.e. by the selection of one alternative arc from each alternative pair of the graph.

The length of entrance and exit due date arcs are defined in the alternative graph as follows. We let i be the first operation of an aircraft in the TCA, γ_i be its scheduled entrance time and q_i be the entrance delay. We assume that the latter information is an input data for the traffic controller. For the landing aircraft, the entrance due date arc has length $-d_i = -(\gamma_i + q_i)$, and the consecutive delay at the entrance of the TCA is $max\{0, t_i - d_i\}$.

The total exit delay is the sum of the *unavoidable delay* (which cannot be recovered by aircraft rescheduling, even if the aircraft travel in the TCA with their minimum travel time) plus the *consecutive delay* (which is required to solve potential conflicts). We let j be the arriving/departing operation at/from a runway r of a landing/take-off aircraft A, δ_j be its scheduled arrival/departing time and τ_j be the earliest possible entrance/exit time in/from the runway r. For each node j of a landing aircraft, τ_j is computed as the sum of the release time plus the minimum travel time into the landing air segments. In case of a departing aircraft, τ_j is the sum of the release time plus the scheduled travel time in the runway. The total exit delay of aircraft A at r is $t_j - \delta_j$. Since we want to minimize the consecutive delay at r, the exit due date arc has length $-d_j = -max\{\tau_j, \delta_j\}$. The unavoidable delay at the runway is $max\{0, \tau_j - \delta_j\}$, while the consecutive delay at the runway is $max\{0, t_j - d_j\}$.

The **MAX TARDINESS** is the formulation that minimizes the maximum consecutive delay both for the entrance and exit due dates, that is the largest deviation from the entrance and due date times due to the resolution of potential conflicts in the TCA during the time horizon considered. This objective function can be represented as makespan minimization by using suitable due date arcs [12, 13]. We observe that all aircraft have the same relevance with this objective function, meaning that this is the most equitable approach. The formulation is next shown:

$$\begin{array}{ll} \min t_n & \\ s.t & \\ t_n - t_k \geq -d_k & \forall (k,n) \in F \\ \{x,t\} \in X & \end{array} \tag{6}$$

The **AVG TARDINESS** is the minimization of the average consecutive delay both for the entrance and exit due dates. The AVG TARDINESS problem is formulated as follows:

$$\begin{array}{ll} \min \frac{1}{|K|} \sum_{k=1}^{|K|} z_k & \\ s.t. & \\ z_k - t_k \geq -d_k & \forall (k,n) \in F \\ z_k \geq 0 & \forall k \in K \\ \{x,t\} \in X & \end{array} \tag{7}$$

where $|K|$ is the number of due date arcs in the alternative graph and z_k is a real variable associated to the due date $k \in K$.

The two objective functions reported above support two different aspects of delay minimization: the maximum tardiness is the most equitable approach (since it minimizes the largest consecutive delay), while the average tardiness has a more global approach (since its value takes into account the delay of all aircraft). We now present a new formulation, named **COMBINED M_A**, that combines the previous objective functions:

$$\begin{array}{ll} \min \frac{\alpha}{\beta} t_n + \frac{(1-\alpha)}{\phi} \sum_{k=1}^{|K|} z_k & \\ s.t. & \\ t_n - t_k \geq -d_k & \forall (k,n) \in F \\ z_k - t_k \geq -d_k & \forall (k,n) \in F \\ z_k \geq 0 & \forall k \in K \\ \{x,t\} \in X & \end{array} \tag{8}$$

where $\beta = t_n^*$, $\phi = \sum_{k=1}^{|K|} z_k^*$ and α is a value between 0 and 1. The latter value is used to balance the importance of each objective function. The values β and $\frac{\phi}{|K|}$ are the best known values for the MAX TARDINESS and AVG TARDINESS problems.

Since the $\alpha = 0$ and $\alpha = 1$ could give arbitrary values for the non-minimized objective, the COMBINED M_A formulation is extended by fixing an upper bound value for the first objective while minimizing the secondary objective. For the $\alpha = 0$ case this constraint is added:

$$t_n \leq \beta \tag{9}$$

For the $\alpha = 1$ case the following constraint is required:

$$\sum_{k=1}^{|K|} z_k \leq \phi \tag{10}$$

The optimal solutions of the *extended* COMBINED M_A formulations are non-dominated solutions only if β and $\frac{\phi}{|K|}$ are the optimal values for MAX TARDINESS and AVG TARDINESS.

The following objective function takes into account the request by some airline companies to limit the number of delayed aircraft above given tolerance thresholds. This corresponds to minimizing the number of deadline violations that often translate into penalty costs, according to service contracts between the airline companies and the traffic control authorities. In our approach, in order to maximize the satisfaction of airline company requirements, all violations have the same wait and we minimize the number of aircraft that have a consecutive delay above a given threshold value $P \geq 0$ at the runways. A violation is thus measured when an aircraft enters the runway with a consecutive delay greater than P. We call this formulation **TARDY JOBS P**:

$$\begin{array}{ll} \min \sum_{f=1}^{U} v_f & \\ s.t. & \\ Mv_f - t_f \geq -d_f - P & \forall (f,n) \in F \\ v \in \{0,1\}^U & \\ \{x,t\} \in X & \end{array} \tag{11}$$

where U is the number of runway operations, f is a runway operation connected with the exit due date d_f, and v_f is a boolean variable indicating if a violation happens (1) or not (0).

We now show a formulation, named **COMBINED M_A_0**, that is a combination of MAX TARDINESS, AVG TARDINESS and TARDY JOBS P, the last one having $P = 0$:

$$\begin{array}{ll} \min \frac{t_n}{\beta} + \frac{1}{\phi}\sum_{k=1}^{|K|} z_k + \frac{1}{\lambda}\sum_{f=1}^{U} v_f & \\ s.t. & \\ t_n - t_k \geq -d_k & \forall (k,n) \in F \\ z_k - t_k \geq -d_k & \forall (k,n) \in F \\ Mv_f - t_f \geq -d_f & \forall (f,n) \in F \\ z_k \geq 0 & \forall k \in K \\ v \in \{0,1\}^U & \\ \{x,t\} \in X & \end{array} \tag{12}$$

where $\lambda = \sum_{f=1}^{U} v_f^*$ is the best known value for the TARDY JOBS P=0 problem.

The best value (possibly the optimum) of each pure objective function is computed beforehand by minimizing each performance indicator, i.e. individually solving the corresponding formulation.

The next two formulations take into account priorities between different types of aircraft. The first formulation is the classical weighted average, named here **PRIORITY TARDINESS**:

$$\begin{array}{ll} \min \frac{1}{|K|}\sum_{k=1}^{|K|} f_k z_k & \\ s.t. & \\ z_k - t_k \geq -d_k & \forall (k,n) \in F \\ z_k \geq 0 & \forall k \in K \\ \{x,t\} \in X & \end{array} \tag{13}$$

where f_k is the weight associated with the due date arc starting from the node k. In this work, the weights are associated to the priority classes described in Section 3.

The second formulation takes into account aircraft priorities and focuses on maximizing the equity between the aircraft of each priority class. Here, the equity is defined as the difference between the largest (D^c_{max}) and the smallest (D^c_{min}) consecutive delays between all aircraft of the same class $c \in C$. This formulation is named **PRIORITY EQUITY**:

$$
\begin{array}{ll}
\min \frac{1}{|C|}\sum_{c=1}^{|C|}(D^c_{max} - D^c_{min}) & \\
s.t. & \\
z_k - t_k \geq -d_k & \forall (k,n) \in F \\
z_k - D^c_{max} \leq 0 & \forall (k,n) \in F \wedge k \in J_c \quad \forall C \\
z_k - D^c_{min} \geq 0 & \forall (k,n) \in F \wedge k \in J_c \quad \forall C \\
D^c_{max} \geq D^c_{min} \geq 0 & \forall C \\
\{x,t\} \in X &
\end{array}
\tag{14}
$$

where $|C|$ is the number of classes and J_c is the set of aircraft belonging to class $c \in C$.

The last group of formulations is related to the maximization of throughput. We consider the formulation **MAX COMPLETION** that minimizes the exit time of the last aircraft from the runway. For this formulation, we fix $d_k = 0\ \forall k \in K$ and we minimize the makespan:

$$
\begin{array}{l}
\min t_n \\
s.t. \\
t_n - t_k \geq 0 \quad \forall (k,n) \in F \\
\{x,t\} \in X
\end{array}
\tag{15}
$$

The **AVG COMPLETION** formulation minimizes the average exit time from the runway:

$$
\begin{array}{ll}
\min \frac{1}{|K|}\sum_{k=1}^{|K|} z_k & \\
z_k - t_k \geq 0 & \forall (k,n) \in F \\
z_k \geq 0 & \forall k \in K \\
\{x,t\} \in X &
\end{array}
\tag{16}
$$

4.3 A numerical example

This section describes a simple traffic situation at the Roma Fiumicino (FCO) TCA, highlighting the difference between the optimal solutions computed for each model of the previous section. For each solution, we provide the value of all the other performance indicators.

Figure 7 presents a schematic view of the TCA and provides the route of each aircraft: A and C are landing aircraft, while B and D are take-off aircraft. A and D are delayed aircraft, i.e. they have an entrance delay that changes their release time. The entrance delay of A is 170 and the one of D is 489. Furthermore, all aircraft have to use the same runway resource (12), since the other one (13) is not available. The presence of disturbances causes potential conflict in the TCA and, therefore, the ASP must be solved. The alternative graph of the traffic situation is shown in APPENDIX A.

Table 1 presents the optimal solution value for each objective function (one per row in bold), and the corresponding value for all other performance indicators. The optimal

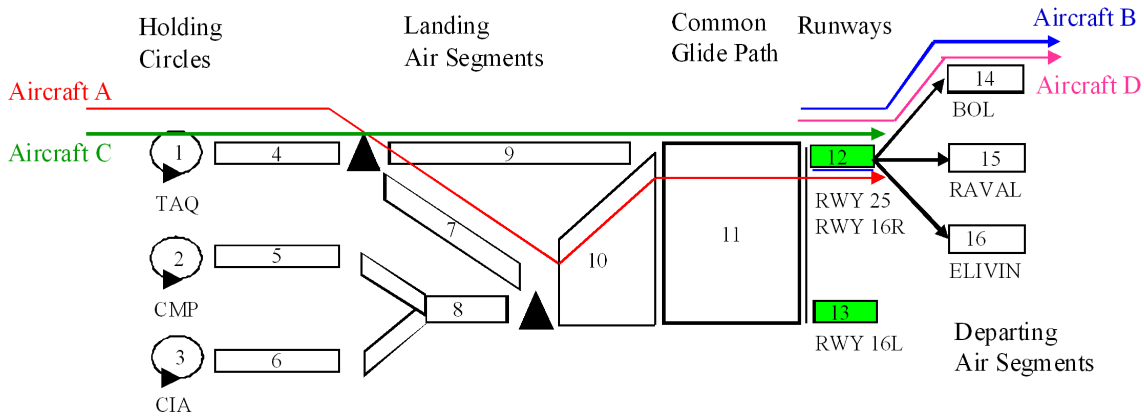

Figure 7: Example situation with landing and take-off aircraft at FCO TCA

solution is obtained by solving the ASP formulations of Section 4.2. Regarding the formulations with aircraft priorities, we consider a different class for each aircraft as described in Section 3. According to the aircraft types and the delayed aircraft, the adopted weights are: $f_A = 20$, $f_C = 10$, $f_D = 2$ and $f_B = 1$. The bigger the weight, the higher the priority class.

Table 1: Optimal ASP solutions computed for each formulation of the numerical example

Objective Function	Tardiness			Completion		Tardy Jobs		Priority
	Max	Avg	Priority	Max	Avg	$P = 0$	$P = 300$	Equity
Max Tardiness	**223**	93.2	913.3	1728	1601	3	0	111.5
Avg Tardiness	225	**68.8**	701.7	1725	1590.5	2	0	103.3
Priority Tardiness	364	85.7	**310.7**	1822	1613	2	1	123.5
Max Completion	225	87.0	1065.0	**1725**	1590.5	2	0	103.3
Avg Completion	225	87.0	1027.5	1725	**1590.5**	2	0	103.3
Tardy Jobs P=0	256	83.8	454.3	1752	1615.3	**2**	0	125.8
Tardy Jobs P=300	256	83.8	497.0	1752	1615.3	2	**0**	125.8
Priority Equity	225	68.8	701.7	1725	1590.5	2	0	**103.3**

From the results of Table 1, we observe that, even for this simple traffic situation with four aircraft, six different solutions are obtained by the eight ASP formulations. Only Avg Tardiness and Priority Equity share the same solution. The different ASP solutions present interesting gaps in terms of the various performance indicators. For instance, looking at the first row of Table 1, Max Tardiness gives the lowest maximum consecutive delay at the cost of large average consecutive delays and number of tardy jobs P=0. In fact, the optimal solution obtained for Max Tardiness minimizes the longest path in the graph, that includes operations of aircraft B and D only. This also explains the high value obtained for Priority Tardiness and Priority Equity (B and D are the two aircraft with lower priorities). Avg Tardiness (second row of Table 1) gives better results than Max Tardiness in terms of average consecutive delays, number of tardy jobs P=0 and Priority Equity, while Priority Tardiness is still far from the optimal value (third row of Table 1). Priority Equity gives a good trade-off between Max Tardiness and Avg Tardiness, however its solution is bad compared to Priority Tardiness. The optimal solution obtained for Priority Tardiness clearly schedules the aircraft on their order of priorities (A, C, D and B), causing a very large consecutive delay for aircraft B (see columns 2 and 8). The values obtained for Max and Avg Completion (fifth and sixth columns of Table 1) are similar for most of the solutions. Tardy Jobs P=0 outperforms Tardy Jobs P=300 and represents a compromise solution in terms of the other indicators. The optimal solution for Tardy Jobs P=0 is shown in APPENDIX B.

5 Experimental results

This section presents the computational results for the ASP formulations of Section 4.2. The tests have been performed in a laboratory environment. We consider real-world ASP instances for FCO and MXP. The ASP solutions are computed via the solver IBM ILOG CPLEX MIP 12.0, fixing the maximum computation time first at 60 seconds and then at 60 minutes. The experiments are executed on a processor Intel Dual Core E6550 (2.33 GHz), 2 GB of RAM and Windows XP.

5.1 Description of the ASP instances

For each terminal control area (MXP and FCO) and time horizon of traffic prediction (30 and 60 minutes), Table 2 describes the 20 ASP instances that we generated with random entrance delays (10 with negative exponential distribution and 10 with Gaussian distribution). In total, the computational study is thus based on 80 ASP instances.

Table 2: ASP instances

TCA	Time Hor. (min)	Landing Aircraft	Departing Aircraft	Max Entr. Delay (sec)	Avg Entr. Delay (sec)	Max Unavoid. Delay (sec)	Avg Unavoid. Delay (sec)
MXP	30	14	6	1792.9	869.6	1476.7	683.5
MXP	60	23	16	1792.9	496.0	1476.7	280.7
FCO	30	16	4	1789.0	990.4	1638.9	783.7
FCO	60	32	16	1789.0	465.0	1638.9	342.9

Each row of Table 2 reports average information over 20 ASP instances, grouped for TCA (FCO and MXP, see Column 1) and time horizon of traffic prediction (30 and 60 minutes, both starting from $t_0 = 0$, see Column 2). Columns 3-4 give the number of landing and departing aircraft, Columns 5-6 the maximum and average entrance delays (in seconds), Columns 7-8 the maximum and average unavoidable delays (in seconds). The latter delays are significantly smaller than the entrance delays, since we compute the free-net travel times by letting all landing aircraft travel with their maximum allowed speed profile.

Table 3 gives the number of variables and constraints for each ASP formulation. Each row reports average information for the 20 ASP instances we considered for each airport (MXP and FCO) and time horizon of traffic prediction (30 and 60 minutes). The FCO instances have more variables and constraints than the MXP instances, since more aircraft are scheduled at FCO and less routing alternatives are available for landing aircraft.

The next subsections will show the computational results obtained for the 80 ASP instances. We tested the 12 ASP formulations: Max and Avg Tardiness dealing with

Table 3: Variables and constraints for each ASP formulation

ASP Instance Size	Number of Variables				Number of Constraints			
TCA	MXP		FCO		MXP		FCO	
Time Horiz (min)	30	60	30	60	30	60	30	60
Max Tardiness	658	1708	806	3222	1307	3397	1605	6425
Avg Tardiness	692	1770	842	3302	1341	3459	1641	6505
Priority Equity	700	1778	850	3310	1409	3583	1713	6665
Priority Tardiness	692	1770	842	3302	1341	3459	1641	6505
Max Completion	658	1708	806	3222	1307	3397	1605	6425
Avg Completion	692	1770	842	3302	1341	3459	1641	6505
Tardy Jobs P=0s	678	1747	826	3270	1327	3436	1625	6473
Tardy Jobs P=300s	678	1747	826	3270	1327	3436	1625	6473

pure delay minimization, Priority Tardiness and Priority Equity dealing with aircraft classes, Max and Avg Completion dealing with throughput minimization, Tardy Job P=0 and Tardy Job P=300 dealing with deadline violations, and the four combined objective functions (Combined M_A_0 and Combined M_A, with/without additional constraints for $\alpha = 0$ and $\alpha = 1$). As described in Section 3, we used four classes of aircraft: 1) landing, delayed aircraft; 2) landing aircraft on time; 3) departing, delayed aircraft; 4) departing aircraft on time. Their weights are: $f_1 = 20$, $f_2 = 10$, $f_3 = 2$ and $f_4 = 1$.

5.2 Pure objective functions

Table 4 presents the average computational results obtained for each ASP formulation with a pure objective function. Each column reports the average results on the 20 ASP instances for a given airport and time horizon of traffic prediction.

For each ASP formulation in Table 4, we have a block of six rows: Row 1 gives the objective function, Row 2 the average computation time (in seconds), Row 3 the number of optimal solutions found by CPLEX within 60 minutes of computation, Row 4/5 the average upper bound (UB) value at 60/3600 seconds (the best value is in bold), Row 6 the average best lower bound (LB) value.

From Table 4, we have the following observations. When comparing the results for the two TCAs, it is often more difficult to compute the optimal solution for Fiumicino airport, since the FCO instances present a larger number of variables and constraints. In particular, CPLEX always reaches the time limit of computation for the 60-minute FCO instances. The results obtained at 60 seconds of computation are similar to the ones obtained at 60 minutes. However, the gap between the best lower and upper bounds is quite large, specially for the 60-minute time horizon.

Regarding the specific performance of the various objective functions, the Tardy Jobs formulations present the larger number of optimal solutions, while Avg Tardiness and Avg Completion the lowest number. More optimal solutions are generally obtained for the objective functions based on a maximum delay minimization compared to the ones based on an average delay minimization. Also, Priority Equity presents a larger number of optimal solutions than Priority Tardiness. The problem of minimizing the maximum consecutive delay, even if this is done per priority class, is thus easier to solve to optimality by CPLEX than the weighted average consecutive delay minimization.

Table 4: ASP solutions for Max Tardiness and Avg Tardiness formulations

TCA	MXP		FCO		MXP		FCO	
Time Horiz (min)	30	60	30	60	30	60	30	60
Obj. Function	**Max Tardiness**				**Avg Tardiness**			
Comp. Time (sec)	454.8	1882.7	197.3	3600	1576.6	3600	1806.2	3600
N. of Optimal Sol.	19	12	20	0	12	0	10	0
UB 60s (sec)	**389.3**	**413.3**	**324.6**	1086.9	**69.0**	69.4	55.0	186.6
UB 3600s (sec)	**389.3**	**413.3**	**324.6**	**1080.5**	**69.0**	**68.7**	**54.6**	**180.0**
LB (sec)	383.8	361.0	324.6	241.4	55.3	20.7	38.0	17.4
Obj. Function	**Priority Equity**				**Priority Tardiness**			
Comp. Time (sec)	11.4	3314.3	77.5	3600	329.5	3600	1651.5	3600
N. of Optimal Sol.	20	2	20	0	20	0	13	0
UB 60s (sec)	**129.4**	226.5	**106.9**	707.0	878.4	803.4	658.4	800.7
UB 3600s (sec)	**129.4**	**224.1**	**106.9**	**658.4**	**878.0**	**790.2**	**652.1**	**756.0**
LB (sec)	129.4	136.5	106.9	112.7	877.9	335.5	577.3	237.1
Obj. Function	**Max Completion**				**Avg Completion**			
Comp. Time (sec)	529.7	1264.3	1508.2	3600	1602.3	3600	1805.0	3600
N. of Optimal Sol.	20	13	13	0	12	0	10	0
UB 60s (sec)	**3363.3**	4248.9	**3485.4**	5284.1	**2463.5**	2925.2	**2670.9**	3561.8
UB 3600s (sec)	**3363.3**	**4245.1**	**3485.4**	**5265.9**	**2463.5**	**2924.2**	**2670.9**	**3550.4**
LB (sec)	3363.3	4220.0	3444.8	4291.0	2439.8	2751.4	2642.2	3184.0
Obj. Function	**Tardy Jobs P = 0s**				**Tardy Jobs P = 300s**			
Comp. Time (sec)	1.3	298.0	18.4	3600	37.6	1212.9	243.6	3600
N. of Optimal Sol.	20	20	20	0	20	15	20	0
UB 60s (sec)	**7.5**	9.2	**8.4**	26.2	**1.5**	2.1	**1.2**	13.9
UB 3600s (sec)	**7.5**	**9.0**	**8.4**	**16.9**	**1.5**	**1.6**	**1.2**	**9.6**
LB (sec)	7.5	9.0	8.4	12.7	1.5	1.2	1.2	0.0

5.3 Optimizing a pure objective and looking at the other objectives

This subsection studies how the optimal ASP solutions computed for a pure objective function infer the quality of the secondary indicators. The proposed analysis permits to assess the quality of non-dominated solutions for one objective function in terms of the other performance indicators. Figure 8 presents eight plots with average results on all the optimal solutions obtained for each pure objective function (51 instances for Max Tardiness, 22 instances for Avg Tardiness, 42 instances for Priority Equity, 33 instances for Priority Tardiness, 46 instances for Max Completion, 22 instances for Avg Completion, 60 Tardy Job P=0s and 55 Tardy Job P=300s). For the set of optimal solutions regarding each pure objective function, Figure 8 gives a plot reporting the percentage gap between the average value of each secondary indicator and the average best known value: (average value − average best value) / average best value. The percentage gap is thus a lower bound on the average difference between the value of each secondary indicator and the corresponding optimal solution.

From Figure 8, Avg Tardiness and Avg Completion are the best compromise solutions, since they are the only objective functions reporting a gap below 100% for all secondary indicators. In general, the objective functions based on a maximum delay minimization present larger gaps compared to the ones based on a average delay minimization (see left versus right plots). Specifically, Avg Tardiness presents a gap of 25% for Max Tardiness and smaller gaps than Max Tardiness for all secondary indicators but Priority Equity. In other words, the average consecutive delay minimization generates less delayed aircraft than the maximum consecutive delay minimization, while the largest delay is considerably higher. The combination of these two objectives is thus worth of investigation.

Regarding the objective functions based on classes and weights, optimizing with priorities does not cause a serious drop of the related pure performance indicators. In fact, Priority Equity presents good values of Max Tardiness, that is the most equitable performance indicator. Similarly, Priority Tardiness gives good values in terms of Avg Tardiness.

When looking at the throughput as a secondary indicator, all the objective functions have performance very close to the best known values of Max Completion and Avg Completion. We conclude that these indicators can easily be taken into account by the studied ASP formulations.

Tardy Jobs are the less equitable objective functions, since they prefer to significantly penalize the behavior of a few aircraft in order to have the set of tardy aircraft as small as possible. Comparing the two corresponding formulations in terms of the other performance indicators, Tardy Jobs P=300sec outperforms Tardy Jobs P=0sec. In general, the latter formulation presents the worst values in terms of the various performance indicators. For this reason, Subsection 5.4 will consider the combination of Tardy Jobs P with other objective functions.

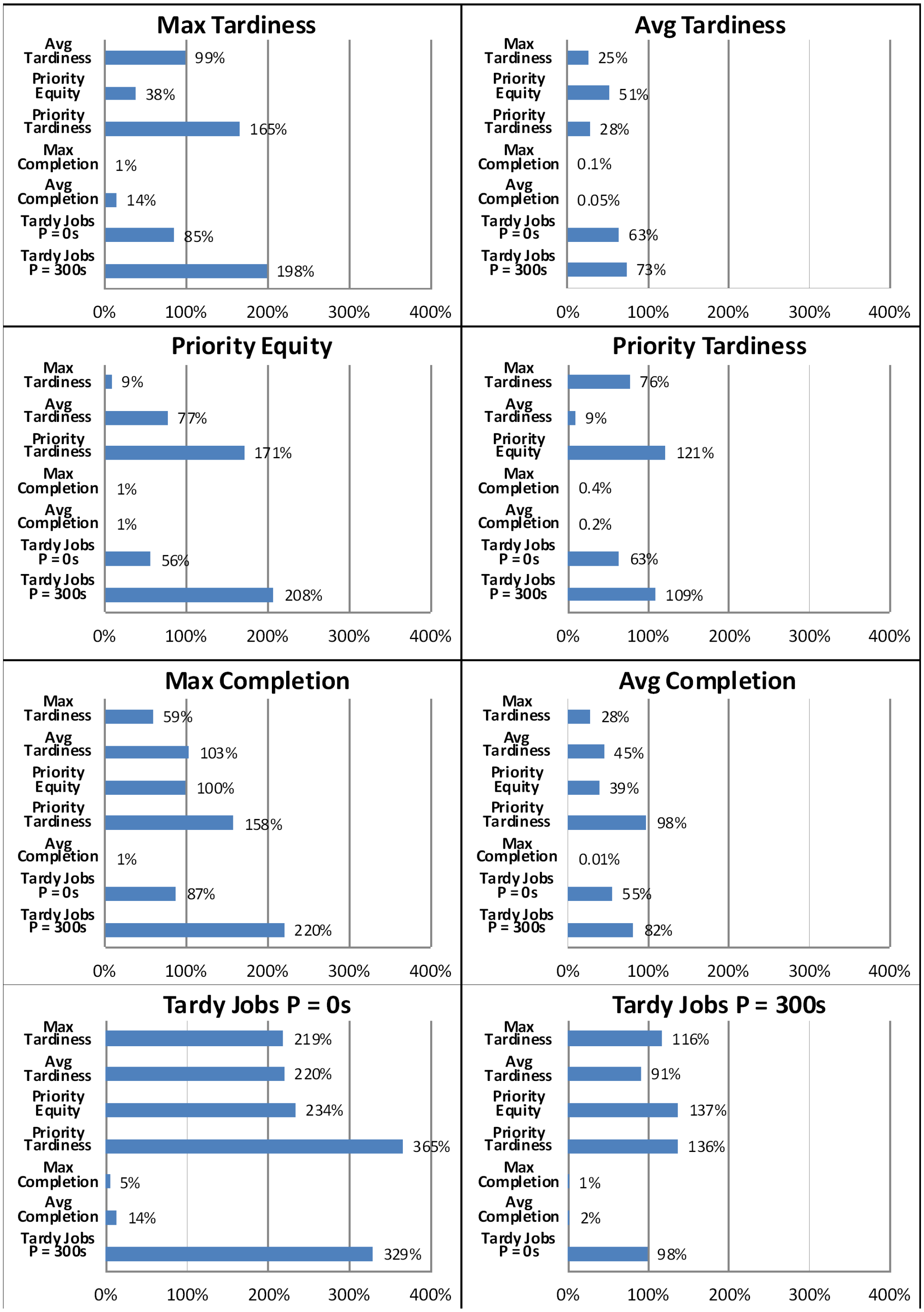

Figure 8: Optimal solutions for pure objectives in terms of the other indicators

5.4 Combined objective functions

This section presents the results obtained on the 80 ASP instances for the combined formulations.

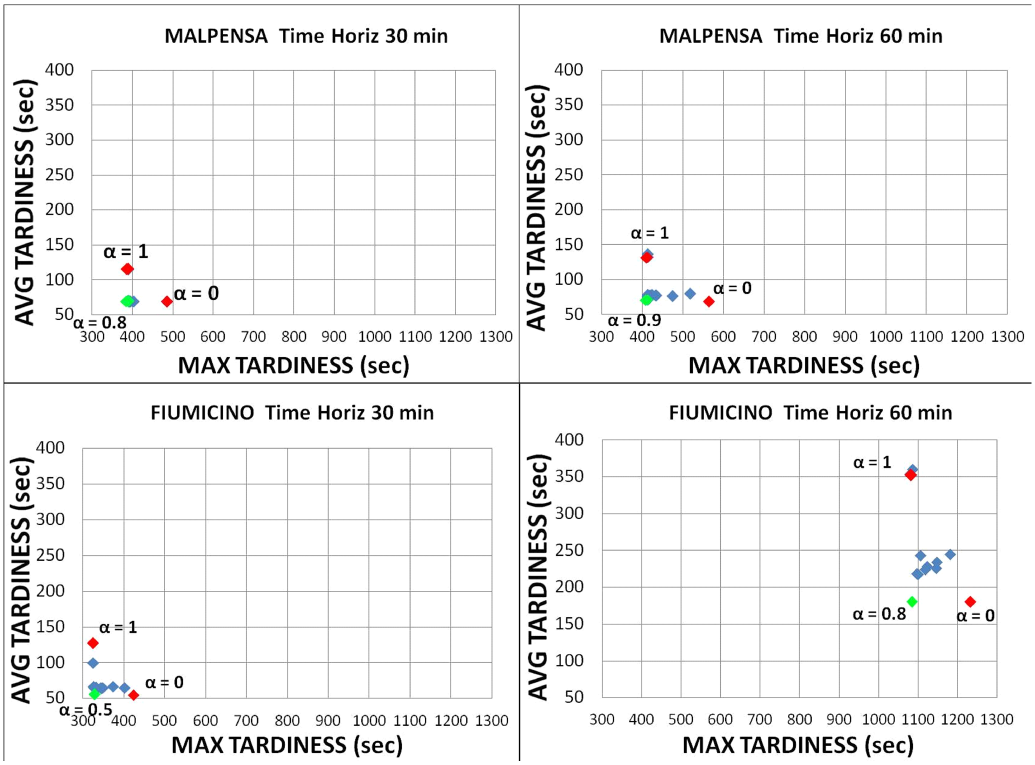

Figure 9: Combined M_A: Max versus Avg Tardiness when varying the value α

Each plot of Figure 9 reports the average results obtained for the Combined M_A formulation and various values of the parameter α on the 20 ASP instances of each airport and time horizon of traffic prediction, in terms of maximum and average tardiness (x-axis and y-axis, in seconds). The objective of this study is to analyze $\alpha \in [0, 0.1, 0.2, ..., 1]$. We considered a time limit of 60 minutes for the extreme values of α (depicted in red), while we used 60 seconds of computation for the other values of α (depicted in blue). Regarding the values β, ϕ and λ, we adopted the best UB values computed by CPLEX with 60-minute computation, as reported in Table 4. For each plot of Figure 9, we consider an *average best value* of α (depicted in green): the point with the minimum distance from the minimum average values obtained for the maximum and average tardiness.

Table 5 shows the results obtained for Combined M_A and Combined M_A_0, with 60 minutes of computation. Each column of Table 5 shows the average results on the 20 ASP instances of each airport (MXP or FCO) and time horizon of traffic prediction (30 or 60 minutes). For the experiments on the Combined M_A formulation, we considered the average best values of α reported in Figure 9 for each airport and time horizon of traffic prediction. Rows 1 gives the average computation time (in seconds), Row 2 the number of optimal solutions for the considered objective function, Rows 3-5 the average values of the three performance indicators considered in Combined M_A_0.

For both objective functions of Table 5, a large number of optimal solutions is obtained for the 30-minute instances, specially for Combined M_A. A few optimal solutions are

Table 5: ASP solutions for Combined M_A and Combined M_A_0

Obj. Function	**Combined M_A**				**Combined M_A_0**			
TCA	MXP		FCO		MXP		FCO	
Time Horiz (min)	30	60	30	60	30	60	30	60
Comp. Time (sec)	542.1	1586.2	99.8	3600.0	1419.9	3582.4	1090.3	3600.0
N. of Optimal Sol.	19	13	20	0	14	1	19	0
Max Tardiness (sec)	389.6	413.3	328.8	1084.4	399.2	453.3	340.4	1118.3
Avg Tardiness (sec)	70.0	70.8	56.3	180.5	78.2	78.5	60.4	239.4
Tardy Jobs P=0s	13.5	21.2	12.6	35.2	10.7	16.6	10.6	24.6

found for the 60-minute instances of MXP and no optimal solution is found for the 60-minute instances of FCO. However, the performance of the combined formulations is, on average, competitive compared to the best known values for each indicator, see Table 4. Overall, the results obtained for Combined M_A outperform the ones obtained for Combined M_A_0, except for Tardy Job P=0s.

Table 6 presents the computational results for the extended Combined M_A formulations by fixing an upper bound for the first objective (the best known UB values β and ϕ are computed by CPLEX with 60-minute computation time) while minimizing the secondary objective. We tested two cases: $\alpha = 1$ in which the maximum tardiness is minimized and the average tardiness is constrained, and $\alpha = 0$ in which the average tardiness is minimized and the maximum tardiness is constrained. For both cases, we set a computation time of 60 minutes for the solver. Each column and row of Table 6 gives the same kind of information reported in Table 5.

Table 6: ASP solutions for the extended Combined M_A with secondary indicators as constraints

Obj. Function	**Extended Combined M_A α=1**				**Extended Combined M_A α=0**			
TCA	MXP		FCO		MXP		FCO	
Time Horiz (min)	30	60	30	60	30	60	30	60
Comp. Time (sec)	1489.5	3600.0	1194.2	3600.0	236.2	1394.6	33.4	3600.0
N. of Optimal Sol.	12	0	14	0	19	15	20	0
Max Tardiness (sec)	444.6	564.2	416.2	1232.2	389.3	413.3	324.6	1080.0
Avg Tardiness (sec)	69.0	68.7	54.6	180.0	70.4	70.9	60.7	183.3

From the results of Table 6, both the extended combined formulations present good average results in terms of maximum and average consecutive delays. Regarding the number of optimal ASP solutions, the maximum tardiness minimization becomes more difficult to solve than the average tardiness minimization when fixing an upper bound to the secondary performance indicator.

Figure 10 shows four plots with average results on all the optimal solutions obtained for each combined objective function (52 instances for Combined M_A, 34 instances for Combined M_A_0, 26 instances for Extended Combined M_A $\alpha = 1$ and 54 instances for Extended Combined M_A $\alpha = 0$). For each combined objective function, Figure 10 gives a plot reporting the percentage gap between the average value of each indicator and the corresponding average best known value.

In Figure 10, all combined objective functions present a good performance for the minimized performance indicators. In general, optimizing a sub-set of indicators generates

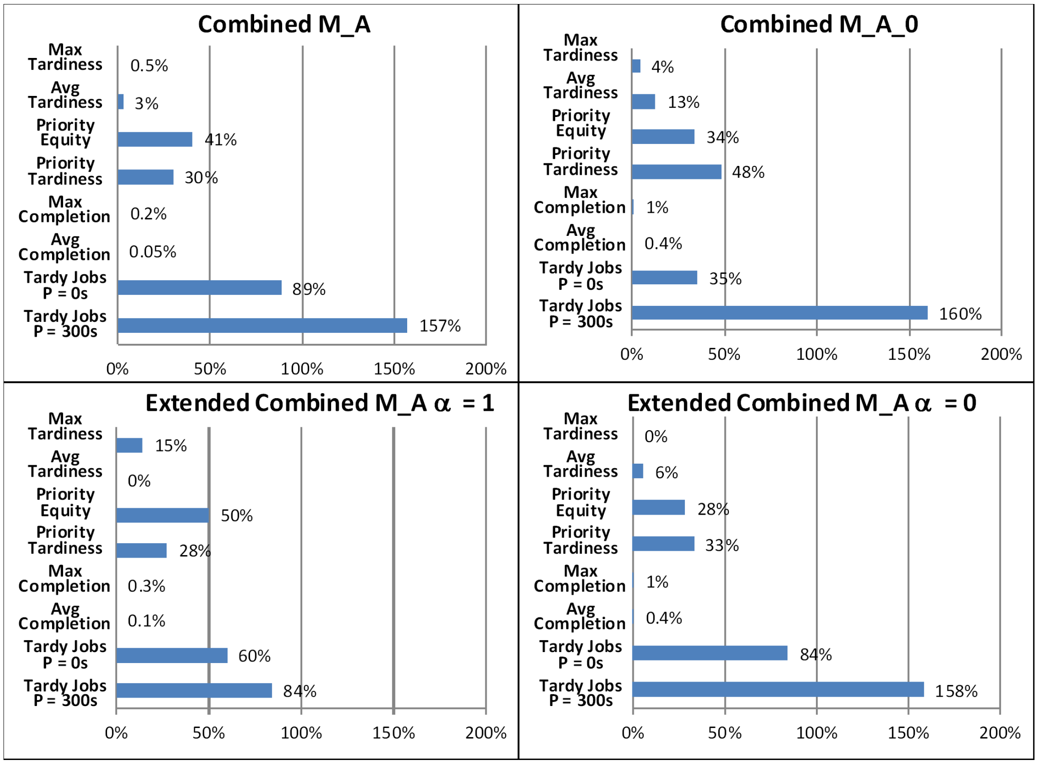

Figure 10: Optimal solutions for the combined objectives in terms of pure indicators

low quality solutions for the other indicators. Good compromise solutions are computed for Extended Combined M_A $\alpha = 1$, reporting a gap below 100% for all secondary indicators. However, there is no combined objective function that outperforms the others for all the indicators.

To summarize, the combination of objective functions results to be a valuable basis for the development of a decision support system, even if a fully informed decision on the ASP solution to be selected for practical realization requires a detailed comparison between the values obtained for each performance indicator and the best known values of each pure objective function.

6 Conclusions and further research

This paper presents microscopic formulations of the ASP with different objective functions of practical interest. A pool of non-dominated solutions is provided to the traffic controller along with a number of quantitative performance indicators related to each ASP solution. An extensive set of computational experiments shows the existence of relevant gaps between the different ASP solutions computed focusing on various aspects of the ASP. A reasonable trade-off between the quality of various performance indicators is found for the ASP solutions computed via combinations of the maximum delay minimization (the most equitable approach) and the average consecutive delay minimization (the most global scheduling approach).

In general, the development of an objective function taking into account all performance indicators is a challenging problem. The solutions resulting from the combination of objective functions may have the drawback to deteriorate the performance related to some of the considered indicators. However, we believe that this work moves the interest of researchers and practitioners in paying more attention to the various performance indicators related to the ASP, and thus on the inherent multi-objective nature of this problem.

Further research will be concentrated on developing new real-time scheduling algorithms for the optimization of pure and combined objective functions that would reduce: 1) the optimality gap found by CPLEX, and 2) the time to compute the best solution. Furthermore, we intend to further improve the microscopic formulations of the ASP and to reduce the quality gaps in terms of the best known values of each performance indicator. The latter result can be achieved by developing new multi-objective ASP formulations, or by introducing additional ASP constraints while optimizing a single indicator (as shown in this paper for the maximum and average tardiness).

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APPENDIX A

The traffic situation of Section 4.3 is modelled by the alternative graph formulation of Figure 11. Each node of the alternative graph represents an operation, e.g. A12 is aircraft A entering runway 12. Black solid arrows represent fixed directed arcs, while coloured dashed arrows represent alternative directed arcs. The length of each arc is depicted in the graph for the routes of Figure 7.

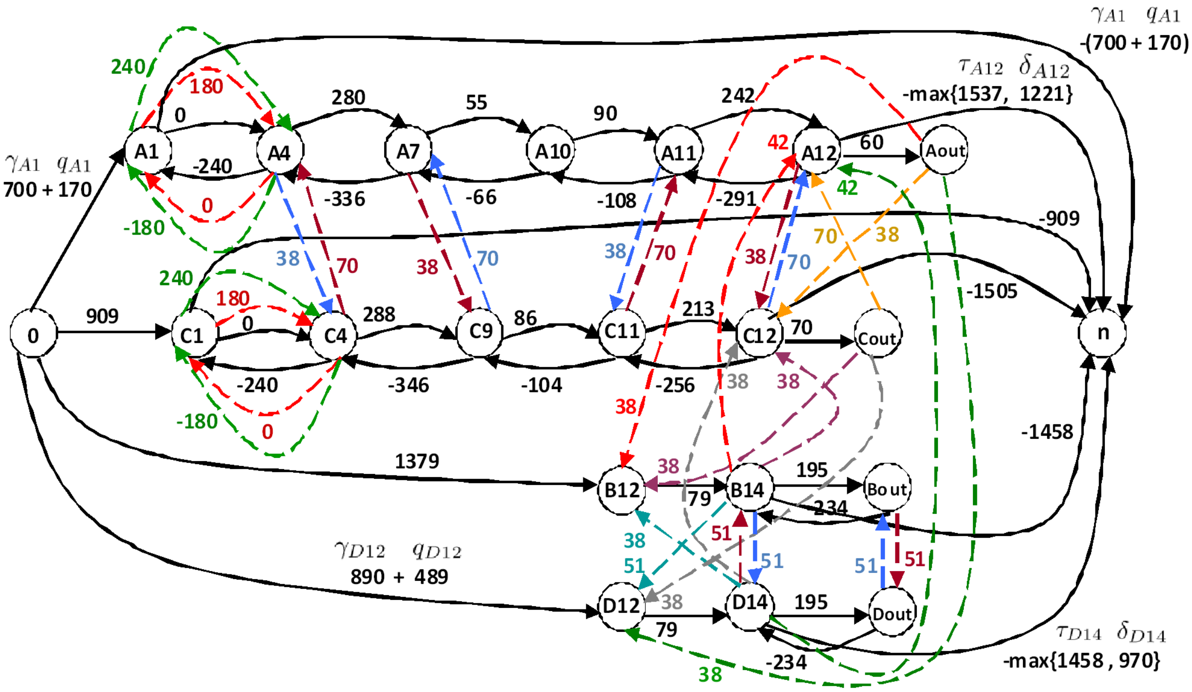

Figure 11: Alternative graph for the numerical example

In this example there are 16 alternative pairs. For the two landing aircraft A and C, we assume that the capacity in the holding circle is unbounded, so there are no potential conflicts between them at the entrance of the TCA. However, we use four alternative pairs (two between nodes A1 and A4 and two between nodes C1 and C4) in order to model the possibility to perform one or two circles in the airborne holding. The shortest circle takes 180 (see the red alternative pairs ((A1,A4),(A4,A1)) and ((C1,C4),(C4,C1))), while the longest 240 (see the green alternative pairs ((A1,A4),(A4,A1)) and ((C1,C4),(C4,C1))). Since aircraft A and C have the same route in the TCA, four alternative pairs model the air segments: the (blue) pair ((A4,C4),(C9,A7)) and the (brown) pair ((C4,A4), (A7,C9)) for air segment 4; the (blue) pair ((A11,C11),(C12,A12)) and the (brown) pair ((C11,A11),(A12,C12)) for air segment 11. Another alternative pair between A and C is required on the runway resource: the (orange) pair ((Aout,C12), (Cout, A12)).

The two take-off aircraft B and D have a potential conflict on the runway resource with the landing aircraft. We thus have to use another five alternative pairs for resource 12: ((Aout,B12), (B14,A12)) (depicted in red), ((Aout,D12), (D14,A12)) (depicted in green), ((B14,C12), (Cout,B12)) (depicted in violet), ((B14,D12), (D14,B12)) (depicted in blue turquoise), ((Cout,D12), (D14,C12)) (depicted in grey). Aircraft B and

D also have a potential conflict on air segment 14, that is modelled by the blue pair ((B14,D14),(Dout,Bout)) and the brown pair ((D14,B14),(Bout,Dout)).

APPENDIX B

Given the example of Section 4.3, Figure 12 shows the optimal solution for the formulation “Tardy Jobs P=0”. The alternative arcs selected in the solution are shown with coloured dashed arrows. The corresponding (complete) selection of the alternative arcs is the following: (A4,C4) and (A7,C9) (aircraft A is scheduled first on air segment 4); (A11,C11) and (A12, C12) (aircraft A is scheduled first on air segment 11); (C1,C4) of length 180 and (C4,C1) of length -180 (aircraft C must perform circles in the holding of length 180); (A4,A1) of length -180 and (A4,A1) of length 0 (aircraft A does not perform circles in the holding); (D14,A12), (D14,B12), (D14,C12), (Aout,B12), (Aout,C12) and (B14,C12) (the runway sequence is D – A – B – C); (D14,B14) and (Dout,Bout) (aircraft D is scheduled first on air segment 14).

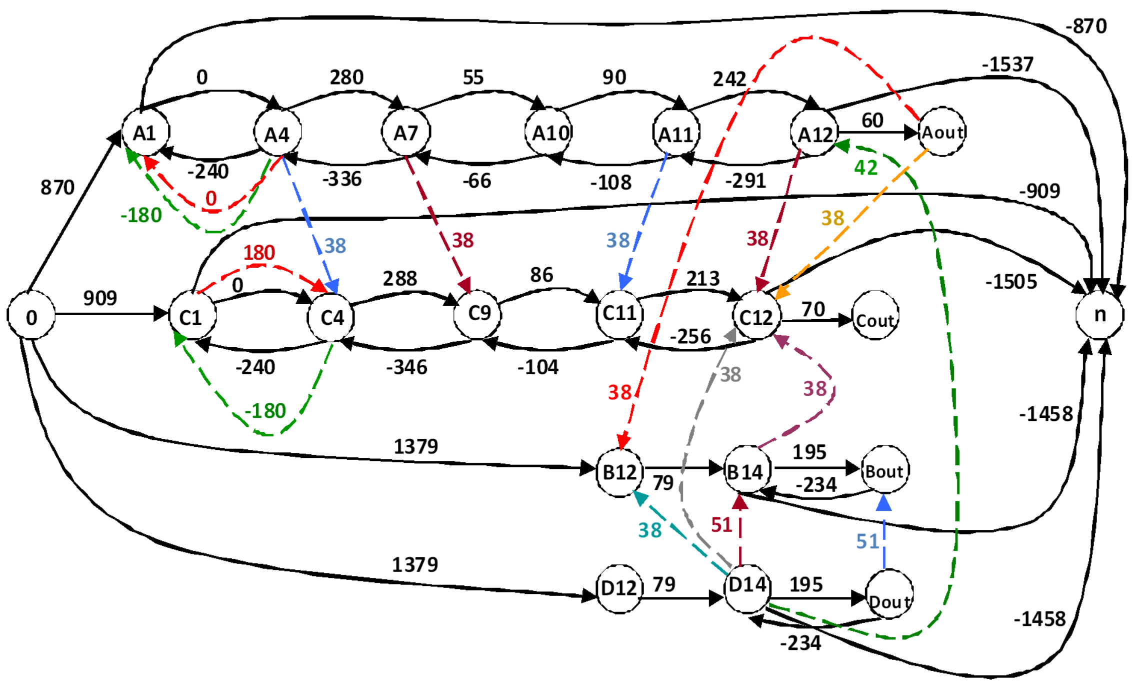

Figure 12: A feasible schedule for the numerical example