A Course on Meta-Heuristic Search Methods for Combinatorial Optimization Problems

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- *Combinatorial optimization problem*: A problem in which an optimal solution is sought among many possible alternative solutions.
- *Heuristic*: A method to discover solution of a problem.
- *Meta-heuristic*: A master strategy that enables a heuristic to adjust the balance between *diversification and intensification* for avoiding a local optimum.

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- *Intensification*: Exploitation of best found solutions to search thoroughly promising regions of the search space.
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why ?

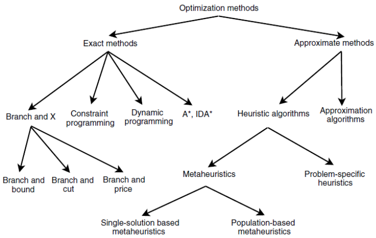

Figure: optimization methods

- It can solve large instances within a reasonable computational time.
- Meta-heuristics are general purpose algorithms.
- Advanced meta-heuristics use search experience.

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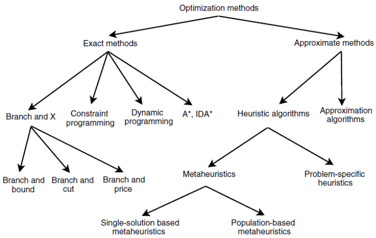

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Working process

step 1:	Generate an initial solution (s)
step 2:	Find $s'(s' \in N(s))$ using a neighborhood operator μ
step 3:	$s \leftarrow s'$ (if s' is better)
step 4:	Repeat *step* 2 - *step* 3 until (*stopping criteria*)

Table: Template of basic local search

Neighbourhood

- More than one solution can be generated in the neighbourhood.
- Acceptance criteria:
  - First improvement
  - Best improvement

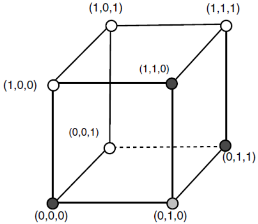


Figure: Cube-shaped neighbourhood

≫ Neighbourhood operator is a systematic mechanism of changing the structure of a solution (e.g., Flip operator for binary strings).

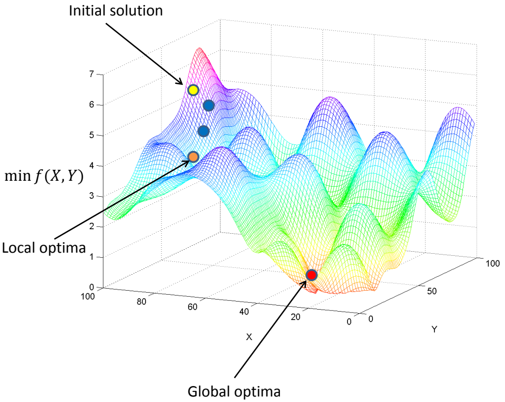


Figure: Search space

Overview

≫ It is a heat treatment process, whereby a metal is heated to a specific temperature and then allowed to cool slowly. The process occurs by the diffusion of atoms and produces a minimum energy crystalline structure.

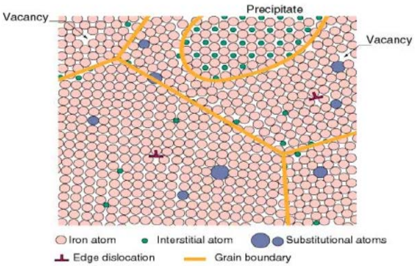


Figure: Microscopic structure of steel

Developed by Kirkpatrick et al. (1983)

Thermodynamics	Simulated annealing
Crystal structure	Solution
Energy	Fitness function
Structure change	Moving to a new solution
Temperature	Control parameter
Equilibrium structure	Best solution

Table: Metaphor

[redacted] Template of simulated annealing algorithm.

Input: Cooling schedule.
$s = s_0$; /* Generation of the initial solution */
$T = T_{max}$; /* Starting temperature */
Repeat
  Repeat /* At a fixed temperature */
    Generate a random neighbor s' ;
    $\Delta E = f(s') - f(s)$;
    If $\Delta E \leq 0$ **Then** $s = s'$ /* Accept the neighbor solution */
    Else Accept s' with a probability $e^{\frac{-\Delta E}{T}}$;
  Until Equilibrium condition
  /* e.g. a given number of iterations executed at each temperature T */
  $T = g(T)$; /* Temperature update */
Until Stopping criteria satisfied /* e.g. $T < T_{min}$ */
Output: Best solution found.

Figure: Pseudocode of Simulated annealing

Temperature effect

≫ Slower is the cooling, better will be the quality of the final solution.

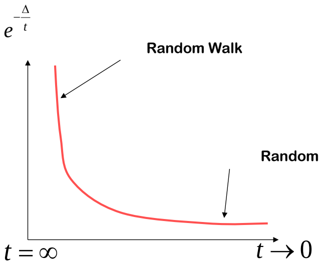


Figure: Effect of temperature

Continue...

Play/Pause

Cooling schedule

- Linear

$$T = T - \beta \qquad \beta; \textit{a constant value} \tag{4.1}$$

- Geometric

$$T = T \times \alpha \qquad \alpha \in [0.5, 0.99] \tag{4.2}$$

- Logarithmic

$$T_i = \frac{T_{initial}}{log(i)} \qquad i : \textit{iteration number}(\textit{outerloop}) \tag{4.3}$$

Continue...

- very slow

$$T_{i+1} = \frac{T_i}{1 + (\beta \times T_i)} \tag{4.4}$$

$$\beta = T - \frac{T_{final}}{(L-1) \times T_{initial} \times T_{final}} \tag{4.5}$$

L: Number of transitions in the inner loop. It should be set according to the size of the problem/neighbourhood size.

Key issues

- Solution representation.
- Initial solution.
- A neighbourhood operator (should generate a valid solution).
- Cooling schedule.
- Length of the inner loop.

Assignment-I

Threshold accepting

Threshold accepting [Dueck and Scheuer (1990)]:

- Acceptance probability

$$P(s', s) = \begin{cases} 1 & \text{if } \triangle E \leq Q_{value} \\ 0 & otherwise \end{cases}$$

- Update Q_{value} according to an annealing schedule.
- Faster than Simulated annealing

Record-to-record

Record-to-record travel algorithm:
RECORD: objective value of the best found solution.

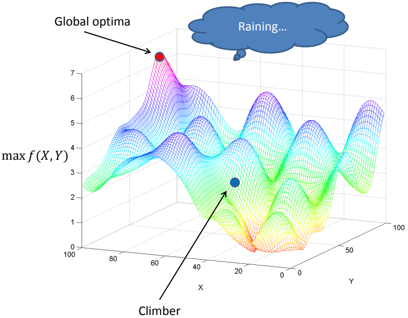
[redacted] Template of the record-to-record travel algorithm.

Input: Deviation $D > 0$.
$s = s_0$; /* Generation of the initial solution */
$RECORD = f(s)$; /* Starting RECORD */
Repeat
  Generate a random neighbor s' ;
  If $f(s') < RECORD + D$ **Then** $s = s'$; /* Accept the neighbor solution */
  If $RECORD > f(s')$ **Then** $RECORD = f(s')$; /* RECORD update */
Until Stopping criteria satisfied
Output: Best solution found.

Great deluge algorithm

Great deluge algorithm [Dueck (1993)]:

- The climber will try to reach at the top (global optima position).
- The climber will try to keep his/her foot above the water level.



Continue...

Input: Water Level;
$s = s_o$ Generation of the initial solution ;
Choose the rain speed UP ;
Choose the initial water level ;
Repeat
Generate a neighbour solution s';
if $f(s') > Level$, then $s = s'$;
Level = Level + UP;
Until (stopping criteria)
Output: Best found solution

Table: Pseudocode of Great deluge algorithm

Dueck, G. (1993). The great deluge algorithm and the record-to-record travel. *Journal of Computational Physics*, 104(1):86–92.

Dueck, G. and Scheuer, T. (1990). Threshold accepting: A general purpose optimization algorithm appearing superior to simulated annealing. *Journal of Computational Physics*, 90:161–175.

Kirkpatrick, S., Gelatt, J., and Vecchi, M. P. (1983). Optimization by simulated annealing. *Science*, 220(4598):671–680.