Tecniche Algoritmiche per Grafi e Reti

Algoritmi approssimati e parametrizzati

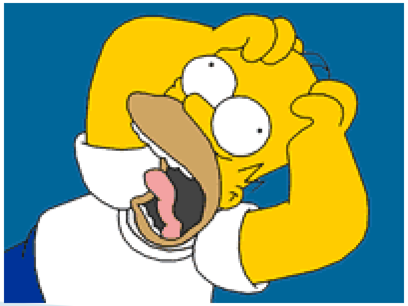
Patrizio Angelini

Dipartimento di Informatica e Automazione
Università degli Studi Roma Tre

A world full of NP-Hard problems

- Problem: I want to travel among a set of cities driving the fewest possible number of KMs.
  - IT'S DIFFICULT!!

- Problem: I want to put a set of objects into some bags, knowing that each bag can not afford more than 10 KGs and trying to use as few bags as possible.
  - IT'S DIFFICULT!!



How to deal with NP-Hard problems?

- **Exact Algorithms**
  - Esatti ma lenti. Solo istanze piccole
- **Approximation Algorithms**
  - Veloci ma non esatti, errore limitato
- **Fixed-Parameter Algorithms**
  - Se un certo parametro è costante, problema polinomiale
- **Randomized Algorithms**
  - Non-determinismo
  - Probabilmente esatti; veloci e probabilmente non molto sbagliati; oppure esatti e probabilmente non troppo lenti

Approximation Algorithms

Approximation Algorithms

- We want a solution that is close to the optimal one
- Given a minimization problem P, an algorithm is an α-approximation for P if, for every instance I of P, it outputs a solution $SOL(I)$ such that

$$SOL(I)/OPT(I) \leq \alpha$$

Approximation Algorithms

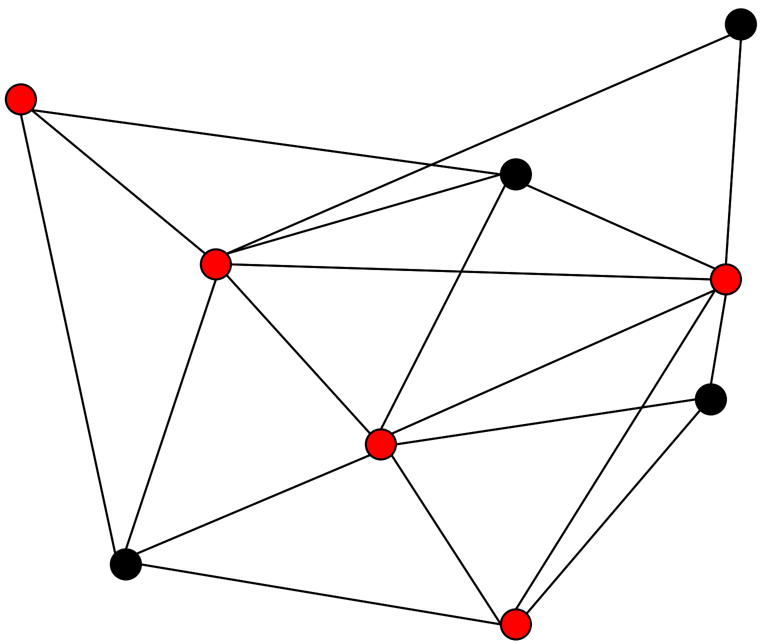
$SOL(I)/OPT(I) \leq \alpha$

If we knew $OPT(I)$ we would not need an approximated solution…

Lower Bounds

Vertex Cover

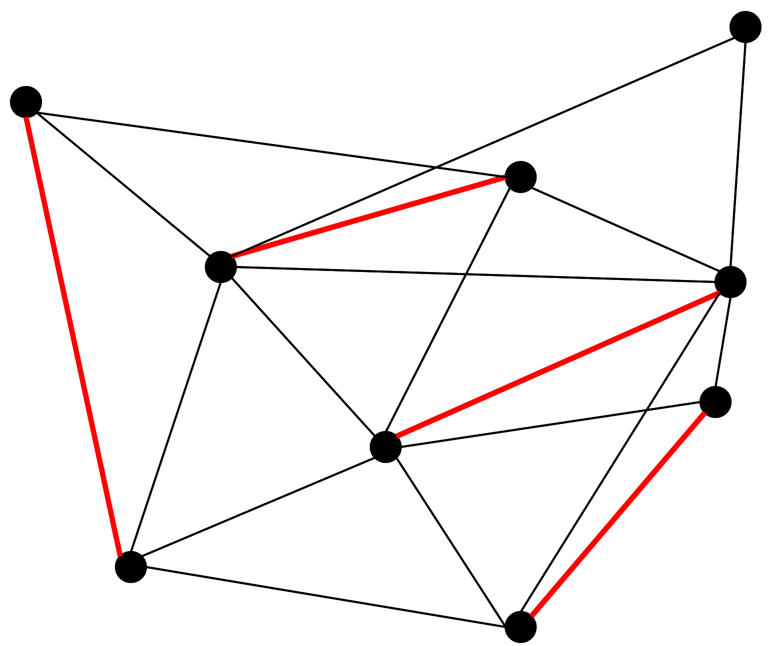
- Problem: find a *minimum vertex cover* of a graph *G(V,E)*, that is, the smallest set $V' \subseteq V$ such that every edge of *E* has an end-vertex in *V'*.



NP-hard ☹
[Karp]

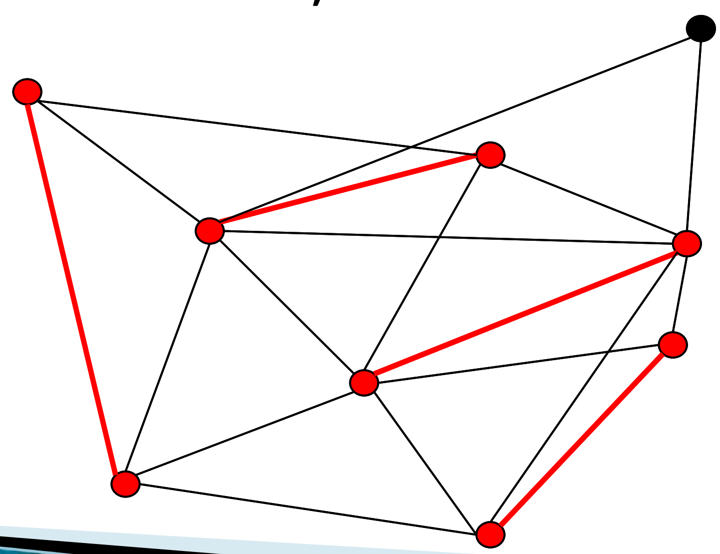
Vertex Cover – Lower Bound

- A *matching* *M* of a graph *G(V,E)* is a set of edges such that no two edges share an end-vertex.
- A matching is *maximal* if no matching *M′* exists such that $M \subset M'$.
- The size of a matching is a lower bound on the size of an optimal solution of *Vertex Cover*



Vertex Cover – Maximal Matching

- ***Lemma**: the set of matched vertices in a maximal matching M of a graph $G(V,E)$ is a vertex cover of G.*
- ***Proof**: Suppose, for a contradiction, that there exists an edge (u,v) such that u and v are not in the vertex cover. Then, u and v are not matched, as well. Hence, (u,v) could be added to M, a contradiction.*



Vertex Cover – Approximation Algorithm

Algorithm: Find a *maximal matching M* of *G* and output the *set S of matched vertices.*

$$SOL(G) = |S| = 2|M| \leq 2OPT(G)$$

$$SOL(G) / OPT(G) \leq 2$$

The above algorithm is a *2-approximation*

Approximation Algorithms: Complexity Classes

INAPX	*Maximum Clique Problem*
APX	*Vertex Cover*
PTAS	*Traveling Salesman Problem* *$(1+\epsilon)$–approx in $O(n^{1/\epsilon})$*
FPTAS	*Knapsack Problem* *$(1+\epsilon)$–approx in $O(2^{1/\epsilon}\, n^{O(1)})$*
P	