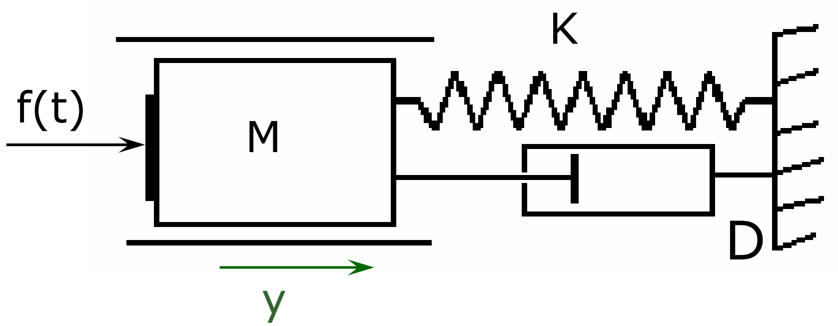
Esempi

Sistema Massa Molla Smorzatore

Posizione del Carrellino



M=1
D=2
K=101

$$f(t) = \delta_{-1}(t), \quad \underbrace{y(0) = 0, \quad \dot{y}(0) = 0}_{Sist.\,a\ riposo}$$

Ovvero:
le sospensioni

$$M\ddot{y}(t) + D\dot{y}(t) + Ky(t) = f(t)$$
$$Ms^2Y(s) + DsY(s) + KY(s) = F(s)$$

Analogo del
sistema R-L-C

$$Y(s) = \frac{1}{Ms^2 + Ds + K} F(s) = \frac{1}{s^2 + 2s + 101} \cdot \frac{1}{s} = \frac{N}{D}$$

$$Y(s) = \frac{1}{Ms^2 + Fs + K} F(s) = \frac{1}{s^2 + 2s + 101} \cdot \frac{1}{s} = \frac{N}{D}$$

$$p_1 = 0 \quad p_2 = -1 - 10j; \quad p_3 = p_2^*$$

$$R_1 = \lim_{s \to 0} s \frac{N}{D} = \frac{1}{101} \to \boxed{\frac{1}{101} \delta_{-1}(t)}$$

$$R_2 = \lim_{s \to -1-10j} (s + 1 + 10j) \cdot \frac{N}{D} = -\frac{1}{202} - \frac{j}{2020}$$

$$R_3 = R_2^*;$$

$$\frac{R_2}{s + 1 + 10j} + \frac{R_3}{s + 1 - 10j} = -\frac{s + 2}{101(s^2 + 2s + 101)}$$

Dobbiamo antitrasformare $$-\frac{s+2}{101(s^2+2s+101)}$$

I poli sono cplx, quindi usiamo

$$e^{-\sigma t}\sin(\omega t)\rightarrow\frac{\omega}{(s+\sigma)^2+\omega^2}=\frac{\omega}{s^2+2\sigma s+(\sigma^2+\omega^2)}$$

$$e^{-\sigma t}\cos(\omega t)\rightarrow\frac{s+\sigma}{s^2+2\sigma s+(\sigma^2+\omega^2)}$$

Da cui: σ = 1, ω = 10 e possiamo risolvere:

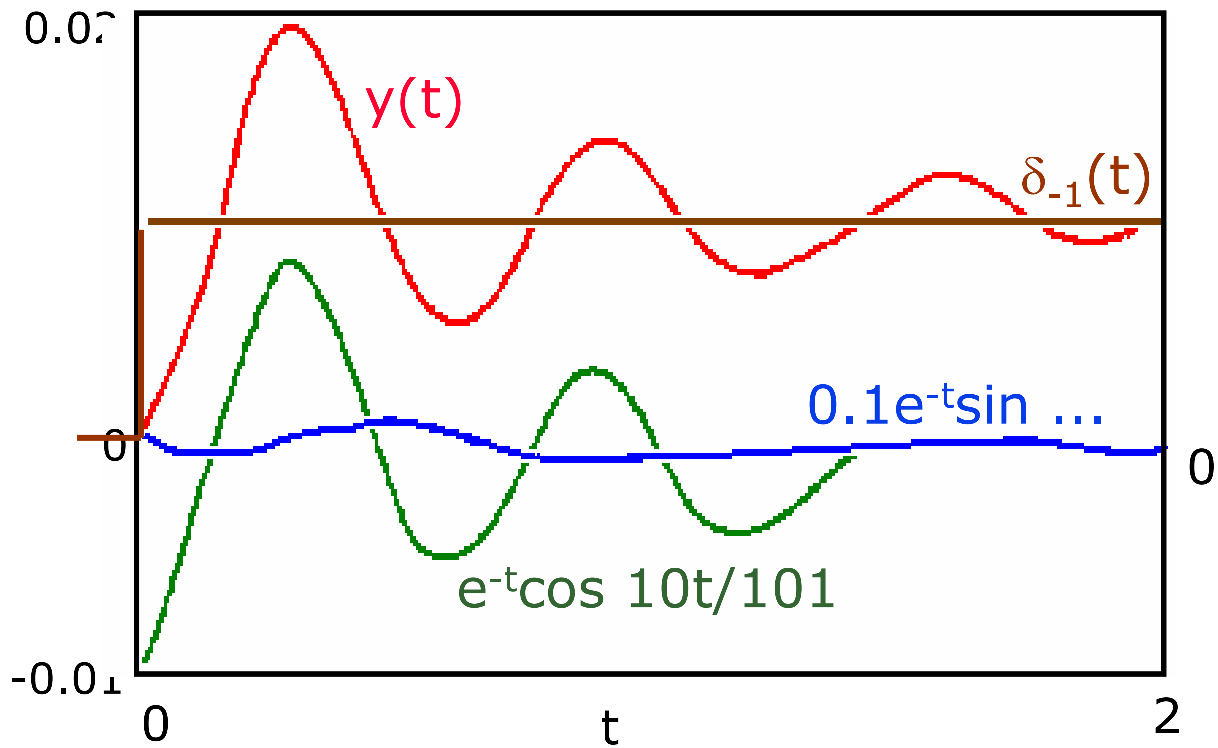
$$A(s+\sigma)+B\omega=-(s+2)\qquad A=-1;\quad B=-0.1$$

E quindi la antitrasformata è $$\left(-e^{-t}\cos 10t-\frac{e^{-t}}{10}\sin 10t\right)$$

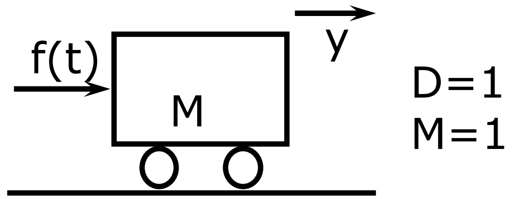
$$y(t) = \frac{1}{101}\left[\delta_{-1}(t) - (e^{-t}\cos 10t - \frac{e^{-t}}{10}\sin 10t)\right]$$

simile all'ingresso

transitorio
legato all'ingresso



$$M\ddot{y} + D\dot{y} = f(t) \qquad y(0) = 0,\ \dot{y}(0) = 0$$



$$f_a = D\dot{x}$$

$$Y(s)\left[Ms^2 + Fs\right] = F(s); \qquad f(t) = \delta_{-1}(t)$$

$$Y(s) = \frac{1}{s(Ms+D)} \cdot \frac{1}{s} = \frac{1}{s^2(Ms+D)} =$$

$$= \frac{R_1^{(1)}}{s} + \frac{R_1^{(2)}}{s^2} + \frac{R_2}{s+1}$$

doppio polo nell'origine

Calcolo dei residui

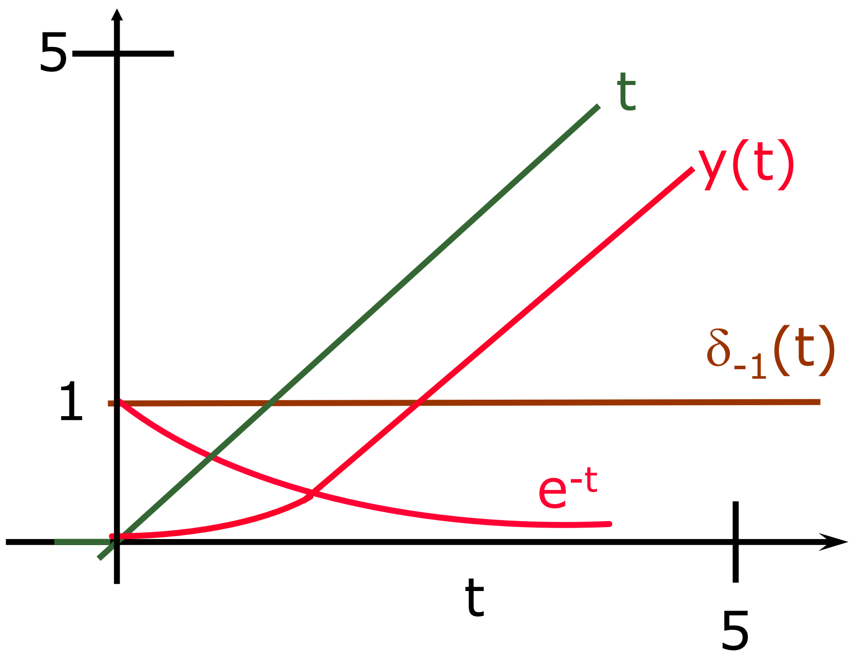
$$R_2 = \lim_{s \to -1} (s+1) \frac{1}{s^2(s+1)} = 1$$

$$R_1^{(2)} = \lim_{s \to 0} s^2 \frac{1}{s^2(s+1)} = 1$$

$$R_1^{(1)} = \lim_{s \to 0} \frac{d}{ds}\left[s^2 \frac{1}{(s+1)s^2}\right] = \lim_{s \to 0} -\frac{1}{(s+1)^2} = -1$$

$$Y(s) = -\frac{1}{s} + \frac{1}{s^2} + \frac{1}{s+1}$$

$$y(t) = +\delta_{-1}(t)\left[-1 + t + e^{-t}\right]$$



Qual'è la parte simile all'ingresso?

Il sistema non è asintoticamente stabile quindi il transitorio non si annulla!