Controllo Digitale
a.a. 2007-2008

Ricostruzione dei segnali

LT-Cap. 3

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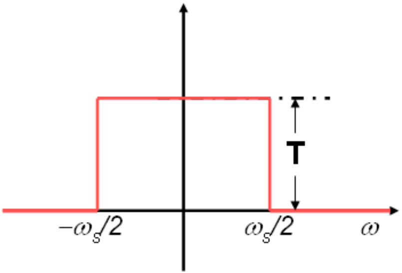
Ricostruttore ideale

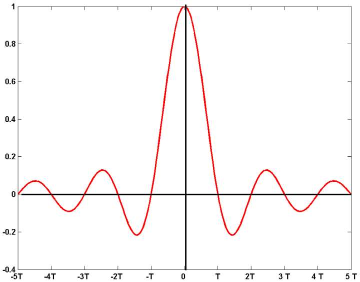
Risposta armonica

$$G_I(j\omega) = \begin{cases} T & -\frac{\omega_S}{2} \le \omega \le \frac{\omega_S}{2} \\ 0 & altrove \end{cases}$$



Risposta impulsiva

$$g_I(t) = \frac{sin(\omega_S t/2)}{\omega_S t/2}$$



Ricostruzione del segnale

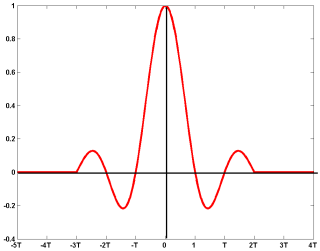
Ricostruzione del segnale con il filtro ideale

$$
\begin{aligned}
x(t) &= \int_{-\infty}^{\infty} x^*(\tau) g_I(t-\tau) d\tau = \\
&= \sum_{-\infty}^{\infty} x(kT) \int_{-\infty}^{\infty} \delta(\tau - kT) \frac{sin(\omega_s(t-\tau)/2)}{\omega_s(t-\tau)/2} d\tau = \\
&= \sum_{k=-\infty}^{\infty} x(kT) \frac{sin(\omega_s(t-kT)/2)}{\omega_s(t-kT)/2}
\end{aligned}
$$

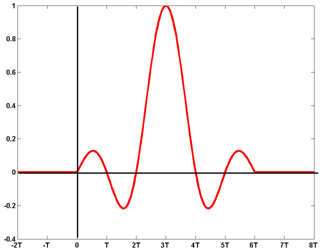
Ricostruttore ideale

Per ricostruire il segnale originario bisogna avere TUTTI ci campioni

Troncamento



Traslazione



Entrambi inaccettabili per
i sistemi di controllo

Ricostruttori reali

Espansione in serie di Taylor

$$x(t) = x(kT) + \left.\frac{dx(t)}{dt}\right|_{t=kT}(t-kT) + \left.\frac{d^2x(t)}{dt^2}\right|_{t=kT}\frac{(t-kT)^2}{2!} + \ldots$$

Derivata=rapp. incrementale

$$\left.\frac{dx(t)}{dt}\right|_{t=kT} \simeq \frac{x(kT)-x((k-1)T)}{T}$$

$$\left.\frac{d^2x(t)}{dt^2}\right|_{t=kT} \simeq \frac{\left.\frac{dx(t)}{dt}\right|_{t=kT} - \left.\frac{dx(t)}{dt}\right|_{t=(k-1)T}}{T} \simeq$$

$$\simeq \frac{x(kT)-2x((k-1)T)+x((k-2)T)}{T^2}$$

Ricostruttore di ordine zero (ZOH)

Segnale ricostruito

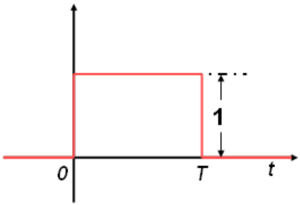
$$x_0(t) = x(kT) \qquad kT \leq t \leq (k+1)T$$

Risposta impulsiva

$$g_0(t) = \delta_{-1}(t) - \delta_{-1}(t-T)$$

$\mathcal{L}$-trasformata

$$H_0(s) = \mathcal{L}[g_0(t)] = \frac{1}{s} - \frac{e^{-sT}}{s} = \frac{1-e^{-sT}}{s}$$



Analisi frequenziale

Risposta armonica

$$\begin{aligned} H_0(j\omega) &= \frac{1-e^{-j\omega T}}{j\omega} = \frac{2e^{-j\omega T/2}}{\omega}\frac{e^{j\omega T/2}-e^{-j\omega T/2}}{2j} = \\ &= \frac{2e^{-j\omega T/2}}{\omega}\sin(\omega T/2) = T\frac{\sin(\omega T/2)}{\omega T/2}e^{-j\omega T/2} \end{aligned}$$

Modulo

$$|H_0(j\omega)| = T\left|\frac{\sin(\omega T/2)}{\omega T/2}\right|$$

Fase

$$\angle H_0(j\omega) = Arg\left[T\frac{\sin(\omega T/2)}{\omega T/2}e^{-j\omega T/2}\right] = Arg\left[\sin\frac{\omega T}{2}\right] - \frac{\omega T}{2}$$

Approssimazione

$$H_0(j\omega) \simeq Te^{-j\omega T/2}$$

Ricostruttore di ordine uno (FOH)

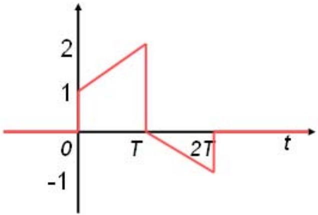
Segnale ricostruito

$$x_1(t) = x(kT) + \frac{x(kT) - x((k-1)T)}{T} \qquad kT \leq t \leq (k+1)T$$

Risposta impulsiva

$$g_1(t) = \delta_{-1}(t) + \frac{\delta_{-2}(t)}{T} - 2\delta_{-1}(t-T) +$$

$$-\frac{2\delta_{-2}(t-T)}{T} + \delta_{-1}(t-2T) + \frac{\delta_{-2}(t-2T)}{T}$$



Ricostruttore di ordine uno (FOH)

$\mathcal{L}$-trasformata

$$
\begin{aligned}
H_1(s) &= \frac{1}{s} + \frac{1}{Ts^2} - 2\frac{e^{-sT}}{s} - 2\frac{e^{-sT}}{Ts^2} + \frac{e^{-2sT}}{s} + \frac{e^{-2sT}}{Ts^2} = \\
&= \frac{1}{s} + \frac{1}{Ts^2}(1 - 2e^{-sT} + e^{-2sT}) = \\
&= \frac{1+Ts}{T}\left(\frac{1-e^{-sT}}{s}\right)^2
\end{aligned}
$$

Analisi frequenziale

Risposta armonica

$$
\begin{aligned}
H_1(j\omega) &= \frac{1+j\omega T}{T}\left(\frac{1-e^{-j\omega T}}{j\omega}\right)^2 = \\
&= T\left(\frac{\sin(\omega T/2)}{\omega T/2}\right)^2 (1+j\omega T)e^{-j\omega T}
\end{aligned}
$$

Modulo

$$
|H_1(j\omega)| = T\left|\frac{\sin(\omega T/2)}{\omega T/2}\right|^2 \sqrt{1+\omega^2 T^2}
$$

Fase

$$
\begin{aligned}
\angle H_1(j\omega) &= Arg\left[T\left(\frac{\sin(\omega T/2)}{\omega T/2}\right)^2 (1+j\omega T)e^{-j\omega T}\right] = \\
&= \arctan(\omega T) - \omega T
\end{aligned}
$$

Ricostruttore a uscita continua

Segnale ricostruito

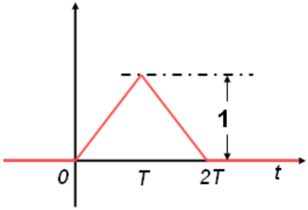
$$x_1(t) = x((k-1)T) + \frac{x(kT) - x((k-1)T)}{T} \quad kT \leq t \leq (k+1)T$$

Risposta impulsiva

$$g_c(t) = \frac{\delta_{-2}(t)}{T} - \frac{\delta_{-2}(t-T)}{T} + \frac{\delta_{-2}(t-2T)}{T}$$

$\mathcal{L}$-trasformata

$$H_c(s) = \frac{1 - 2e^{-sT} + e^{-2sT}}{Ts^2} = \frac{1}{T}\left(\frac{1-e^{-sT}}{s}\right)^2$$



Analisi frequenziale

BUON

LAVORO