Controllo Digitale

a.a. 2007-2008

Mapping $s \longrightarrow z$

LT-Cap. 3

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Legame $s \rightarrow z$

Data una funzione $x(t)$ campionata

$$X^*(s) = X(z)\Big|_{z=e^{sT}}$$

da cui si deduce i legame $s \rightarrow z$

$$z = e^{sT}$$

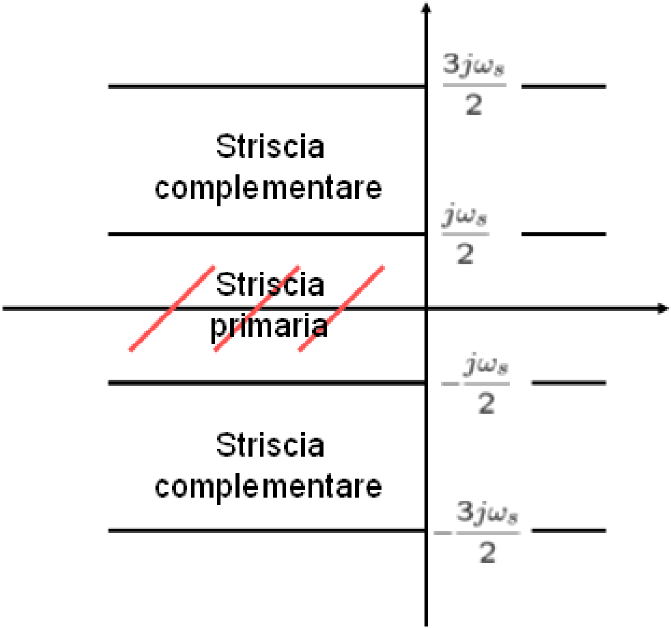
s è una variabile complessa

$$s = \sigma + j\omega$$

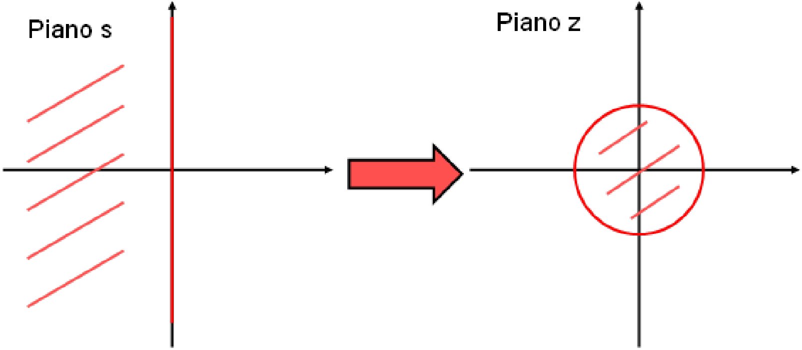
da cui

$$z = e^{sT} = e^{T(\sigma + j\omega)} = e^{T\sigma} e^{jT(\omega + \frac{2k\pi}{T})} \quad \forall k \in \mathbb{Z}^+$$

Suddivisione piano S



Poli stabili



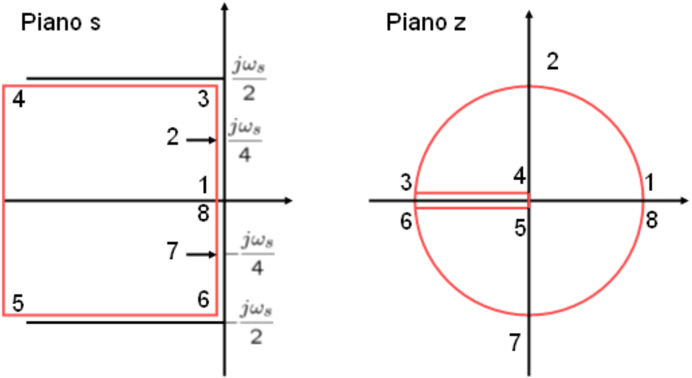
Asse immaginario

$$z = e^{0T} e^{j\omega T}$$

Modulo

$$|z| = e^{T\sigma} < 1$$

Polo nell'origine (1-8)



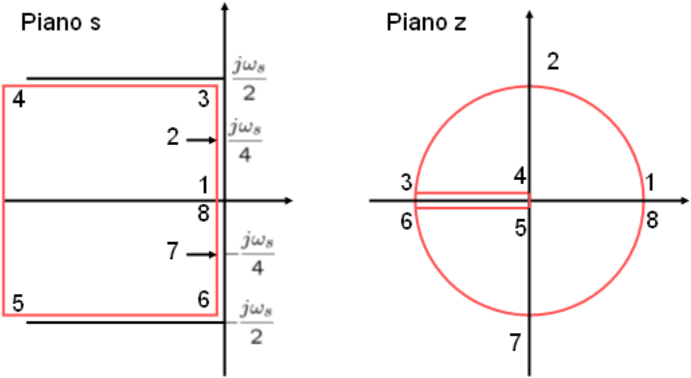
Modulo

$$|z| = e^{0T} = 1$$

Fase

$$\angle z = \angle e^{j0^- T} = \angle e^{j0^+ T} = 0$$

Poli sull'asse immaginario (2-7)



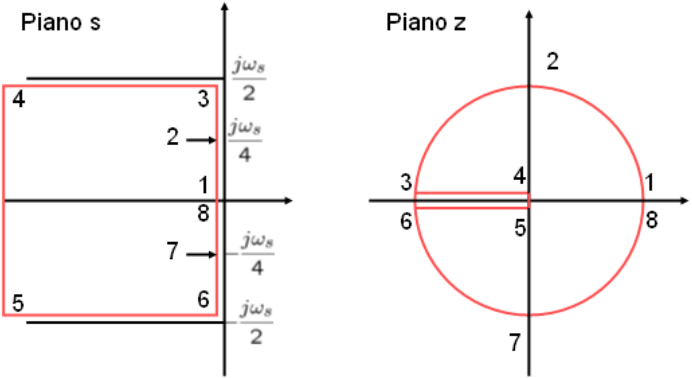
Modulo

$$|z| = e^{0T} = 1$$

Fase

$$\angle z = \angle e^{\frac{j\omega_S T}{4}} = \angle e^{\frac{j2\pi T}{4T}} = \frac{\pi}{2}$$

Poli al confine della striscia primaria (3-6)



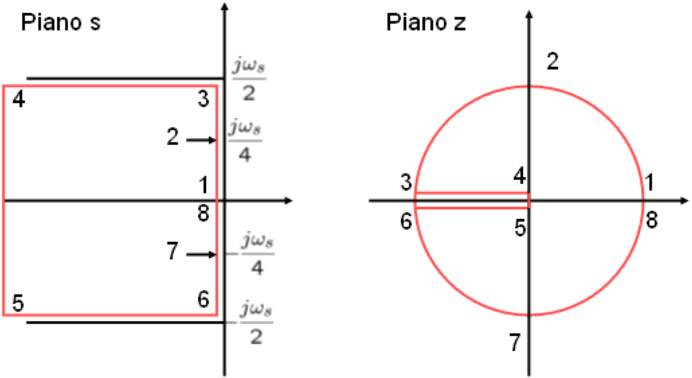
Modulo

$$|z| = e^{0T} = 1$$

Fase

$$\angle z = \angle e^{\frac{j\omega_S T}{2}} = \angle e^{\frac{j2\pi T}{2T}} = \pi$$

Poli al confine della striscia primaria (4-5)



Modulo

$$|z| = e^{-\infty T} = 0$$

Fase

$$\angle z = \angle e^{\frac{j\omega_S T}{2}} = \angle e^{\frac{j2\pi T}{2T}} = \pi$$